From vector-chain-based to geometrical Tolerance Analyses of Systems in Motion – the Use of Meta-models and their Evaluation

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Abstract

Statistical tolerance analyses are usually performed to ensure the functional capabilities of a product, which underlies deviations. Therefore, several tolerance analysis techniques are available, whereas the vector-chain-based tolerance analysis is well-known and widely used. However, this technique is limited concerning the consideration of geometrical deviations, like the deformation of parts. This paper presents a methodology, which overcomes this limitation by the use of meta-models. These meta-models are used to represent the deviations as well as the effects of other deviations towards those (interactions). Since meta-models are approximations, a certain error appears. Hence, the accuracy of a tolerance analysis depends largely on the meta-model's prediction quality. Consequently, three different case studies are performed, based on a non-ideal one-way clutch, to answer the question on "the prediction quality the meta-models should have".

Keywords: Tolerance analysis, interactions between deviations, system in motion, non-ideal mechanism, meta-model, Coefficient of Prognosis COP

1 Introduction

Dimensional and geometric deviations heavily affect a product, its characteristics and the behavior during every stage of the product's lifecycle. In order to ensure the capabilities (e.g., concerning manufacture, assembly or functionality) of the product, the product developer has to consider the appearing deviations as early as possible. Therefore, usually tolerance analyses are used in the predictive engineering [1]. Several different tolerance analysis theories can be used: M-Space-Theory

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[2], Technologically and Topologically Related Surfaces (TTRS) [3], vector-chain-based tolerance analysis (e.g., [4]) and geometry-based tolerance analysis (e.g., using the skin-model-approach [5]).

This paper focuses on the statistical tolerance analysis of a system in motion using vector-chains. The consideration of mechanism results in three major challenges:

- The time-dependence of the motion behavior and thus, the system's functional key characteristics [6]
- The appearance of additional kinds of deviations (e.g., operation-depending deviations which appear during the system's use) [1,6]
- The interactions between different kinds of deviations, which cause additional variations of the system's functional key characteristics [4,7] (Figure 1)

The presented methodology faces these challenges by specific modifications of the tolerance analysis' vector-chain. These modifications include the use of meta-modeling techniques to represent the operation-depending deviations. These meta-models are mathematical models which can be easily integrated into the vector-chain. Moreover, also the effects of appearing interactions between deviations can be taken into account by means of these meta-models. However, the generation of these meta-models requires the determination of the considered operation-depending deviations, which is usually done using (geometry-based) CAX-Tools. Consequently, the consideration of systems in motion using a vector-chain based tolerance analysis requires a step towards the geometrical tolerance analysis. The paper both details the modified methodology as well as focuses on the necessary use of the meta-models.

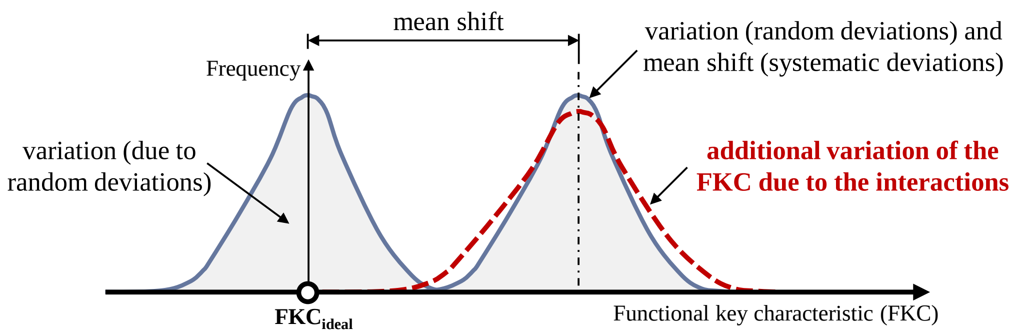

Figure 1: Effects on a functional key characteristic (FKC), caused by random and systematic deviations as well as the interactions between deviations

2 State of the art

The two objectives of the tolerance management (tolerance analysis and tolerance synthesis) are well known and widely used today. However, the existing tolerance analysis-approaches and –methodologies are not taking into account the challenges of a mechanism during its use. In this context, HASENKAMP [8] notices, in considering the entire lifecycle of a product, that the development of integrated methods is a promising and essential aim in research on tolerance management and robust design.

2.1 Vector-chain-based tolerance analysis of systems in motion

The parts of a mechanism underlie different kinds of deviations, which appear in different stages of the product's lifecycle. These deviations have effects on the time-dependent functional key characteristics of the mechanism: Manufacturing-caused deviations can be traced back to manufacturing discrepancies. The effects of these deviations are considered in several publications concerning mechanisms with lower [9,10] as well as higher kinematic pairs [11]. Operation-depending deviations appear during the product's use. The operation-depending relative displacement between a mechanism's parts due to joint clearance is considered in [12], while IMANI [13] integrates elastic deformations into the tolerance analysis of a mechanism. Further publications consider mechanisms with both manufacturing and operation-depending deviations (e.g., [14]).

However, the previously detailed publications do not take into account the time-dependence of the mechanisms and thus, of the deviations. The "integrated tolerance analysis of systems in motion" [1,6] is taking this step. The methodology enables the product developer to investigate the effects of manufacturing-caused and operation-depending deviations on time-dependent functional key characteristics of systems in motion. The visualization of the results of the "integrated tolerance analysis of systems in motion" is detailed in [15].

In summary, the presented research on tolerance analysis of mechanism focuses on the effects of manufacturing-caused deviations and/or operation-depending deviations on the functional key characteristics of technical systems. However, possible interactions between deviations and the effects on the functional key characteristics have not yet been taken into account.

2.2 Use of meta-modeling techniques in tolerance management

As long as ten years ago, HONG [16] stated, that meta-modeling techniques like artificial neural networks (ANNs) can pave the way to "a systematic method which automates this procedure, incorporating the domain specific knowledge as well as the geometry and process knowledge". The use of meta-models in tolerance-related investigations is still very limited – despite this auspicious potential. However, an extended use of meta-models in tolerance management could be noticed in recent years [17].

The use of meta-models in tolerance management can be separated into two main applications: On the one hand, the mathematical relations between the appearing deviations and the functional key characteristics are formulated/approximated by means of meta-modeling techniques. These relations are needed to perform statistical tolerance analyses of a non-ideal system. For instance, SCHLEICH [18] uses the Response Surface Methodology (RSM) to approximate the deformation of a flexible beam in bending, while WATRIN [19] approximates the relation between the noise level of a car's rear axle bevel gear and the manufacturing-caused deviations of the gear parts. Aside of the widely used RSM, several additional meta-modeling techniques can be found in tolerance-related publications. The failure probability of a non-ideal car-door assembly is predicted by means of the Support Vector Regression in [20]. Moreover, the effects of manufacturing deviations of a non-ideal assembly can be represented by artificial neural networks, as shown in [21]. On the other hand, meta-models can replace the relations between appearing tolerances and the resulting manufacturing costs. These tolerance-cost-relations are needed in case of tolerance synthesis, but usually unknown. Consequently, the approximation of the dependence between the component's tolerances and the resulting costs is considered in several publications, using, e.g., the RSM [22] or ANNs [23].

3 Vector-chain-based vs. geometrical tolerance analysis

Basically, the statistical tolerance analysis can be divided into three main steps: First, a mathematical relation, which describes the dependencies between the appearing deviations and the system's functional key characteristic, is required. The second step includes the application of the tolerance analysis method (e.g., Worst-Case-Analysis or Monte-Carlo-Simulation), based on the functional relation. According to the chosen method, a destined number of non-ideal systems (samples) are generated virtually. The determination of the functional key characteristic of each of these non-ideal systems closes the second step of the tolerance analysis. The representation and interpretation of the tolerance analysis' results (usually: FKC's probability distribution and contributors) is the closing third step. [4,6]

The different tolerance analysis theories can be essentially distinguished by the way, the functional relations are established. The vector-chain-based tolerance analysis uses closed vector-chains, which include both the appearing deviations and the system's FKC. Therefore, the appearing dimensional and geometrical deviations must be reduced to plain vectors. Consequently, information about the shape and the volume as well as the corresponding deviations of the system's components cannot be taken into account entirely.

The geometrical tolerance analysis does not require the vectorial reduction of the surface and volume information. So, the geometrical tolerance analysis can consider both dimensional as well as geometrical deviations (e.g. shape deviations like flatness) entirely. Moreover, also operation-depending deviations (like deformation or thermal expansion) can be integrated into the tolerance analysis, since usually geometrical information of the system's parts are necessary to determine these deviations. However, the formulation of the functional relations (needed for the geometrical tolerance analyses) is highly complicated and currently still subject of numerous research activities. Furthermore, the computational effort increases rapidly, when using the geometrical instead of the vector-chain based tolerance analysis.

Consequently, the product developer has to deal with two diverging tolerance analysis-options: Vector-chain-based tolerance analysis (quick and simple, but simplified) vs. geometrical tolerance analysis (time-consuming and complex, but more realistic.

4 Vector-chain-based tolerance analysis of non-ideal systems in motion

The “integrated tolerance analysis of systems in motion” [1,4,6] enables the product developer to investigate non-ideal systems in motion, using a vector-chain-based tolerance analysis. This methodology already faced two challenges which are indicated above: The time-dependence of the system and the consideration of operation-depending deviations. First, the functional relation between the system’s FKCs and the appearing random manufacturing-caused deviations $\mathrm{Dev_i}$ must be formulated using time-dependent terms, according to:

$$FKC(t) = f[Dev_1(t), Dev_2(t), \dots, Dev_i(t), t] \tag{1}$$

The consideration of operation-depending deviations causes that the classic vector-chains are reaching its limits, since usually geometrical information of the parts is needed to determine these deviations. Consequently, the integration of operation-depending deviations requires a step towards the geometrical tolerance analysis. Therefore, at first additional vectors are integrated into the functional relation and the vector-chain, respectively. These additional vectors represent the deterministic operation-depending deviations. [4,6]

However, the detailed modifications do not allow the consideration of interactions between the appearing deviations during a tolerance analysis of a non-ideal mechanism. Consequently, two additional modifications of the existing tolerance analysis methodology are necessary:

- Interactions towards random deviations can be taken into account during the second step of the tolerance analysis – the tolerance analysis method. If a Monte-Carlo-based sampling method (e.g., Latin-Hypercube Sampling) is used, these interactions can be easily taken into account by correcting the sampling’s covariance matrix C (entries aside the principal diagonal).
- The consideration of interactions towards systematic (or deterministic) deviations is much more complex, since those must be taken into account during the formulation of the functional relation (first step of the tolerance analysis), too. Consequently, an additional mathematical model is required, which represents the systematic deviations as well as the corresponding interactions towards this deviation. Therefore, meta-models will be used.

The procedure to generate/train the necessary meta-models consists of four steps, as detailed in Figure 2. At first, a destined number of samples of the non-ideal mechanism must be generated. Therefore, several sampling-techniques (associated to the Design of Experiments; DoE) can be used. However, due to the high computational and time expense of the following CAX-simulations, a Latin-Hypercube Sampling (LHS) is preferred, which requires far less samples to combine plausibly varying parameter values according to their individual distribution.

The second step includes the determination of the deviations which are affected by interactions for each of the generated samples from the previous step. Therefore, both analytical equations as well as numerical simulations can be used –

depending on the considered deviation and the desired accuracy. However, since several operation-depending deviations of mechanisms are caused by forces, usually multi-body-dynamics simulations (MBD) are used. For instance, the time-dependent deformation of a mechanism's components can be investigated using a flexible MBD (coupling of MBD and Finite-Element-Analysis).

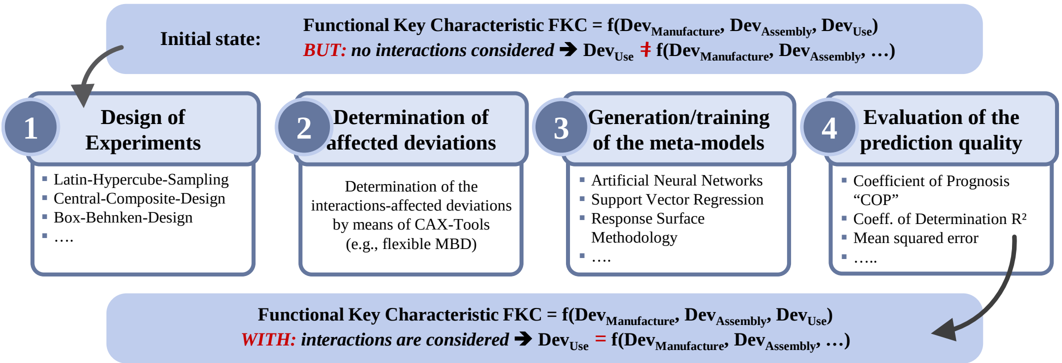

Figure 2: Procedure to generate/train a meta-model, representing the systematic deviations and the corresponding interactions towards this deviation

Based on the statistical CAX-simulations of the generated non-ideal mechanism-samples, the resulting data-set can be used to generate/train the meta-model in the following step. Therefore, in addition to the well-known Response Surface Methodology (RSM), several additional meta-modeling techniques can be used. As detailed in section 2.2, tolerance-related researchers applied inter alia the Support Vector Regression (SVR) as well as Artificial Neural Networks (ANNs).
The data-set is divided into two sets – the so-called training-set and the test-set. While the training-set is used to generate/train the meta-model, the remaining test samples are needed to evaluate the prediction quality of the meta-model. To obtain the training and test sets, several strategies are available. Those validation strategies differ in the way the data-set is split into the training and test-sets. Usually a Split-Validation with the commonly used split ratio of 70:30 (training:testing) is used [7].
Finally, the prediction quality of the meta-model has to be evaluated. Therefore, so-called "goodness-of-fit"-parameters (e.g., mean square error, Coefficient of Determination R^2, and Coefficient of Prognosis COP [24]) can be used. If a satisfying prediction quality could be achieved, the meta-model can be integrated into the tolerance analysis' functional relation, replacing the terms of the considered deviations, which underlie interactions. Subsequently, the product developer can proceed to the specific next step of the statistical tolerance analysis of the mechanism – the application of the tolerance analysis method.
In conclusion, by means of the presented methodology, the product developer is able to perform a vector-chain-based statistical tolerance analysis of a system in motion, which underlies different kinds of dimensional and/or geometric deviations (e.g., manufacturing-caused and operation-depending deviations). Furthermore, the interactions between the appearing deviations can be taken into account by means of appropriate meta-models.

5 Evaluation of meta-models using the COP

The use of meta-models allows the consideration of interactions as well as a significant reduction in computational and time expense of statistical tolerance analyses of mechanisms. However, since these meta-models are just approximations of deviations and the interactions towards these deviations, the reliability of a tolerance analysis depends largely on the meta-models' prediction qualities. Consequently, the product developer has to answer the question, which prediction qualities a meta-model should have to ensure the tolerance analysis' accuracy.
In order to support the product developer, appropriate recommendations should be derived in the following sub-section, based on three case-studies of a non-ideal one-way clutch. Hereby, the Coefficient of Prognosis

$$COP = \left(\frac{E(Y_{test} \cdot Y_{train})}{\sigma_{Y(test)} \cdot \sigma_{Y(train)}}\right)^2 \tag{2}$$

is used to evaluate the prediction quality, whereas the standard deviations σ of the sample distributions Y_{test} and Y_{train}, and their means E are required [24]. The COP ranges between 0 and 1 and a COP of 0.9 is equal to prediction quality of 90 % [24]. The studies' demonstrator is presented in the upcoming sub-section 5.1, while the three case studies are detailed in 5.2 – 5.4.

5.1 Demonstrator: One-way clutch

The one-way clutch assembly is a well know and widely used demonstrator in tolerance-related publications, stretching back to 1967 [25]. The clutch transmits a torque in a single rotational direction. The assembly consists of four main components: gear shaft, outer ring (bearing race) as well as four balls, which are constrained by four springs. According to Figure 3, the four springs are compressed by the balls in case of a clockwise rotation of the gear shaft. A counter-clockwise rotation of the gear shaft leads to a compression of the springs by the four balls. Consequently, no torque is transmitted between the gear shaft and the outer ring, if a counter clockwise rotation of the gear shaft occurs.

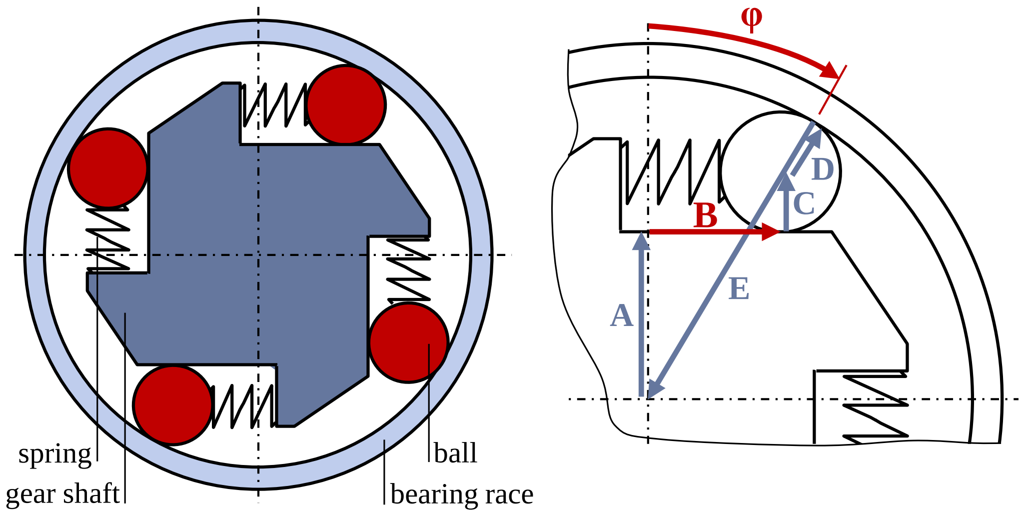

Figure 3: One-way clutch: Assembly and vector-chain [26]

The assembly's functional capabilities depend largely on the pressure angle φ. It furthermore defines the dimension B, which correlates to the horizontal position of the balls. Hence, the clutch's functional key characteristics are the pressure angle φ as well as the dimension B, which depend on the clutch's four dimensions A, C, D, and E [27]:

$$\varphi = \arccos\left(\frac{A+C}{E-D}\right) \tag{3}$$

$$B = \mathrm{E}\cdot\sin(\varphi) - \mathrm{D}\cdot\sin(\varphi) \tag{4}$$

5.2 Study #1: "The worst meta-model is no meta-model"

In order to derive recommendations for the product developer on an appropriate evaluation of a meta-model's prediction quality, the first case study is focusing on the lower limit of acceptable COPs.

Therefore, the following hypothesis will be investigated: *The prediction quality of a meta-model (which predicts certain values) has to be higher than the prediction quality, which can be achieved if the values are assumed randomly.* So consequently, the first case study can be designated "The worst meta-model is no meta-model".

The study includes a total of six investigations with an increasing number of values N which are assumed randomly (N = 100, 200, 400, 600, 800, and 1000). It is assumed, that the random assumptions have the same distribution as the reference; a 6σ-gaussian distribution with 10±2. The COPs of each investigation were determined, using a 10-fold Cross Validation [28], where 10 % of the samples were used to determine the COP. Consequently, ten COPs are shown in Figure 4 for each investigation.

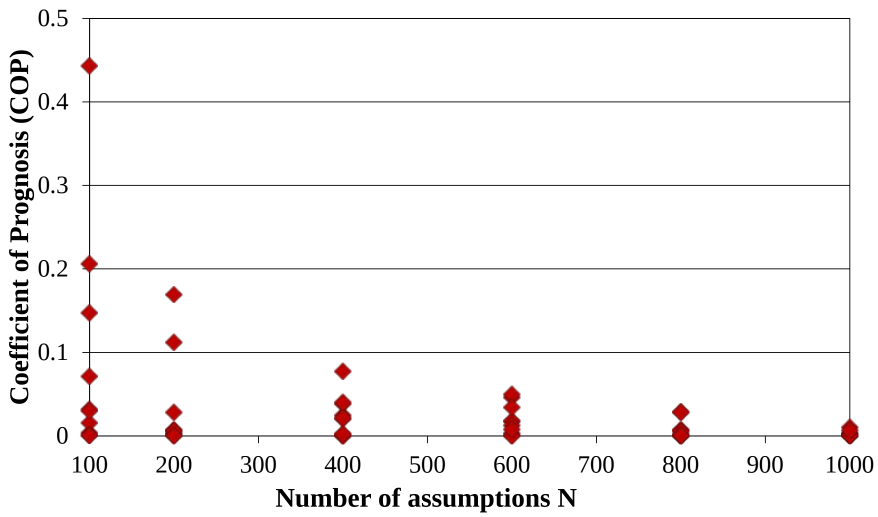

Figure 4: COPs of the random assumptions for six investigations with varying number N of assumptions

The results show, that the lower the number of assumptions gets, the higher the COP can be. This indicates, that especially with a low number of samples, COPs of up to ~0.5 can be achieved just by random assumptions.

5.3 Study #2: The impact of variation on the COP

Since the COPs increases, the more the meta-model's predicted values differ from its desired reference values. The second study considers the impact of the predicted values' variation on the corresponding COP. The procedure of these investigations is shown in Figure 5.

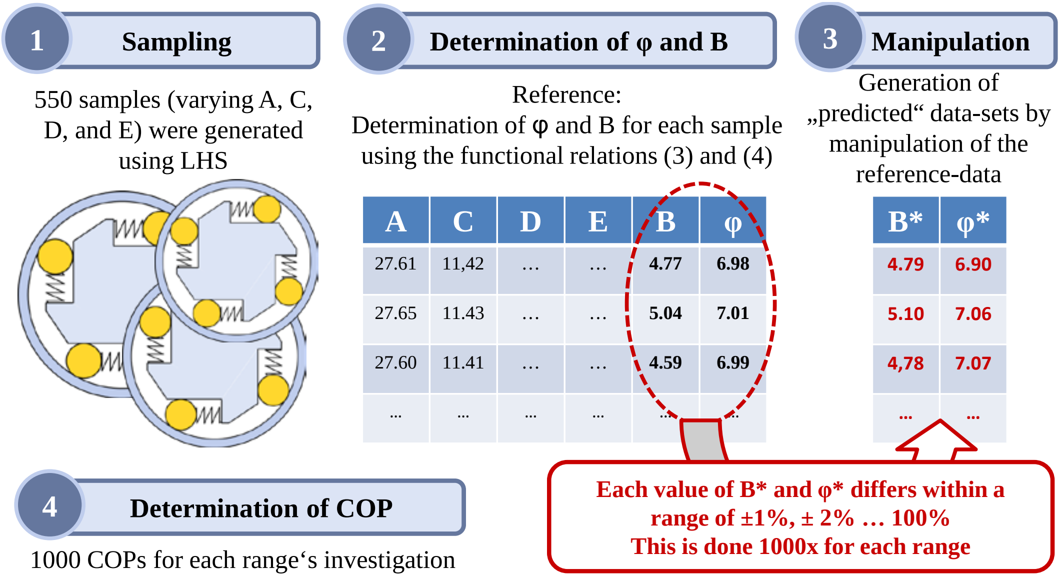

Figure 5: Procedure of case study #2

Therefore, 550 virtual one-way clutches with varying dimensions A, C, D, and E are generated, using a Latin-Hypercube Sampling. The corresponding reference FKCs φ and B of each clutch-sample are to be determined using the equations (3) and (4). Consequently, these values correspond to a COP of 1, which means a prediction quality of 100 %.

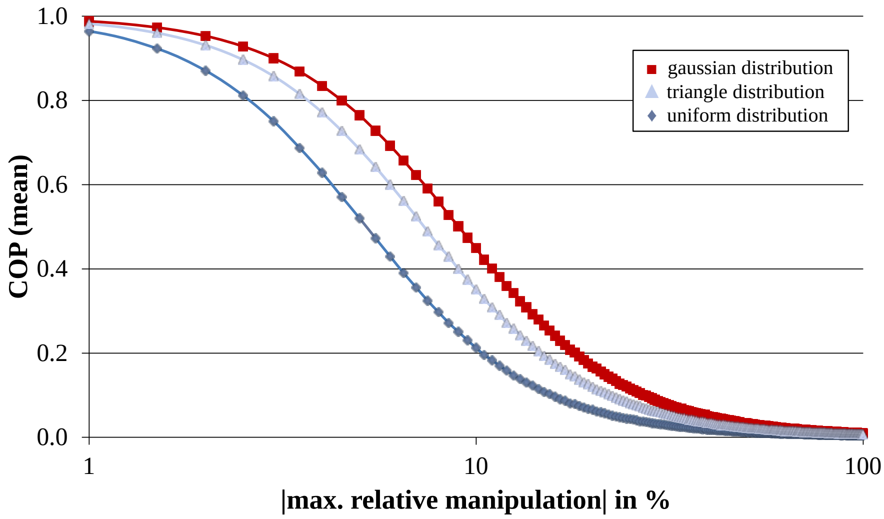

Figure 5: COPs of the random assumptions for six investigations with varying number N of assumptions

In order to investigate the impact of variation of the predicted values (concerning the reference data-set) on the COP, each sample's FKCs φ and B are manipulated on purpose – but within a certain range. These manipulations are done several times, starting with a range of ±1%. Moreover, the manipulations were assumed to correspond to a) gaussian, b) triangle, and c) uniform distribution. The determination of the COP is done 1000 times for each manipulation as well as varying distribution. The resulting means of the COP of φ are shown in Figure 5, whereas the abscissa details the maximum relative error of the manipulations. Based on these results, a quite sensitive behavior of the COPs towards variation could be identified.

5.4 Study #3: The impact of statistical outliers on the COP

The final case study bases on the 550 clutch-samples from the previous study. However, aside of the effect of variation on the COP (study #2), also the impact of statistical outliers on a meta-model's prediction quality is investigated. Therefore, additional outliers are included in the already manipulated data-sets of study #2. These outliers are varying within the range of a relative error of ±40 %. Moreover, the percentage of samples, which underlie relative errors of max ±40 % (statistical outliers), is increased constantly. Similar to the second study, the COP is determined 1000 times, to ensure reliable statistical results. Three essential results of this study are detailed in Table 1.

Table 1: Three investigations on the impact of outliers on the COP

COP (mean of 1000 runs)	**Max. relative error of all 550 samples (uniform distributed)**	**Percentage of outliers of the 550 samples (uniform distributed)**
0.98	0.5 %	0.0 %
0.80	0.5 %	0.5 % (3 of 550 samples)
0.50	0.5 %	2.0 % (11 of 550 samples)

The results show, that the COP is very sensitive towards the appearance and amount of statistical outliers. While a relative error of 0.5 % of each samples FKC results in a COP of 0.98, already a very small amount of statistical outliers (3 of 550 samples = 0.5 %) leads to a rapidly decreasing COP to ~ 0.80. Moreover, with an in increasing percentage of statistical outliers (11 of 550 samples = 2 %), the COP drops drastically to 0.50.

6 Recommendations on the interpretation of the COP

As, detailed in section 4, the integration of geometrical deviations in vector-chain-based tolerance analyses can be achieved by means of meta-models. However, since meta-models are just approximations of the appearing deviations and the interactions towards these deviations, the reliability of the tolerance analysis depends largely on the meta-models' prediction qualities. Based on the results of the three studies, three essential recommendations can be derived. These recommendations aim to support the product developer on answering the question, which prediction qualities a meta-model should have to ensure a reliable and efficient statistical tolerance analysis of a non-ideal system in motion.

- Meta-models with a COP < 0.5 are not reliable, since these COPs can also be achieved by random assumptions. Consequently, the use of these meta-models is highly critical.
- Meta-models with COPs < 0.8 are very likely to predict values which differ significantly from the "real" value (statistical outliers). The use of these meta-models must be critically questioned and additional experiments (samples) may be required to generate/train a more precise meta-model.
- Meta-models with COPs > 0.8 are likely to predict a very number of low statistical outliers.

However, especially in case of 6σ-investigations, the meta-model's prediction qualities should be as high as possible (COPs > 0.98), since even a small prediction error can cause significant varying results and thus, problems in manufacturing, assembly and use of the considered systems in motion.

Acknowledgment

The authors would like to thank Vitalij Schuller, B.Sc. for his support on the three case studies. Furthermore, we thank the German Research Foundation (DFG) for supporting the research project ME1029/16-1 "Functional product validation and optimization of technical systems in motion as a part of product lifecycle oriented tolerance management".

References

[1] WARTZACK, S. ; MEERKAMM, H. ; STOCKINGER, A. ET AL.: Lebenszyklusorientierte Toleranzsimulation zur funktionalen und ästhetischen Produktabsicherung. In: *Konstruktion* 6 (2011). S. 63-67.

[2] TURNER, J.U.: Tolerances in computer-aided geometric design. In: *The Visual Computer December* 3 (1987), S. 214-226.

[3] ZHANG, H-C.: *Advanced Tolerancing Techniques*. New York, USA : John Wiley & Sons, 1st ed., 1997.

[4] WALTER, M. ; WARTZACK, S.: Analysis of the effects of manufacturing-caused deviations and varying operation parameters on operation-depending deviations of systems in motion. In: *Proceedings of the 12th CIRP Conference on Computer Aided Tolerancing*. Huddersfield, United Kingdom, April 2012, paper no. 55.

[5] SCHLEICH, B. ; ANWER, N. ; MATHIEU, L. ; WALTER, M. ; WARTZACK, S.: A Comprehensive Framework for Skin Model Simulation. In: *Proceedings of the ASME 11th Biennial Conference On Engineering Systems Design And Analysis 2012*. Nantes, France, July 2012.

[6] STUPPY, J. ; MEERKAMM, H.: Tolerance analysis of mechanisms taking into account joints with clearance and elastic deformations. In: *Proceedings of the 17th International Conference on Engineering Design*. Stanford, USA, August 2009. S. 488-500.

[7] WALTER, M. ; SPRÜGEL, T. ; WARTZACK, S.: Tolerance analysis of mechanism taking into account the interactions between deviations using meta-models. In: *Proceedings of the 9th Norddesign Conference*. Aalborg, Demark, August 2012. S. 587-594.

[8] HASENKAMP, T. ; ARVIDSSON, M. ; GREMYR, I.: A review of practices for robust design methodology. In: *Journal of Engineering Design* 6 (2009), S. 645-657.

[9] ADABI M.E. ; FARKHONDEH, S. ; FARKHONDEH, F.: Tolerance Analysis of Mechanisms by Using a Fictitious Slider. In: *European Journal of Scientific Research* 3 (2010), S. 385-398.

[10] GERMER, C.: Statistische Toleranzanalyse von beweglichen Karosserienanbauteilen. In: *Konstruktion* 7 (2010), S. 44-47.

[11] BRUYÈRE, J. ; DANTAN, J.Y. ; BIGOT, R. ET AL.: Statistical tolerance analysis of bevel gear by tooth contact analysis and Monte Carlo simulation. *Mechanism and Machine Theory* 42 (2007), S. 1326-1351.

[12] SACKS, E. ; JOSKOWICZ, L.: Parametric kinematic tolerance analysis of general planar systems. *Computer-Aided Design* 30 (1998), S. 707-714.

[13] IMANI, B.M. ; POUR, M.: Tolerance analysis of flexible kinematic mechanism using DLM method. *Mechanism and Machine Theory* 44 (2009), S. 445-456.

[14] HANZAKI, A.R. ; RAO, P.V.M. ; SAHA, S.K.: Kinematic and sensitivity analysis and optimization of planar rack-on-pinion steering linkages. *Mechanism and Machine Theory* 44 (2009), S. 42-56.

[15] STUPPY, J. ; MEERKAMM, H.: Statistical tolerance analysis and result visualisation for systems in motion. In: *Proceedings of the 11th International Design Conference*. Dubrovnik, Croatia, May 2010, S. 1431-1440.

[16] HONG, S.Y. ; CHANG, T.C.: A comprehensive review of tolerancing research. In: *International Journal of Production Research* 11(2002), S. 2425-2459.

[17] DANTAN, J.Y. ; GAYTON, N. ; DUMAS, A. ET AL.: Mathematical Issues in Mechanical Tolerance Analysis. In: *Proceedings of the 13th Colloque National AIP PRIMEC*. Le Mont Dore, France, 2012, S. 1-12.

[18] SCHLEICH, B. ; STOCKINGER, A. ; WARTZACK, S.: On the impact of geometric deviations on structural performance. In: *Proceedings of the 12th CIRP Confer-

ence on Computer Aided Tolerancing. Huddersfield, United Kingdom, April 2012.

[19] WATRIN, S. ; BINZ, H. ; VAN DE LINDELOOF, B.: Methodology for requirement-driven tolerance specification of bevel gears. In: *Proceedings of the 12th CIRP Conference on Computer Aided Tolerancing*. Huddersfield, United Kingdom, April 2012.

[20] WANG, H. ; YU, T.X. ; PANG, H. ET AL.: Mechanism Reliability Simulation Analysis for Multi-Support Axis Seizure Considered Assembly Tolerance. In: *Proceedings of the International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering*. Xi'an, China, 2011. S. 35-41.

[21] ANDOLFATTO, L. ; THIÉBAUT, F. ; DOUILLY, M. ET AL.: On neural networks' ability to approximate geometrical variation propagation in assembly. In: *Proceedings of the 12th CIRP Conference on Computer Aided Tolerancing*. Huddersfield, United Kingdom, April 2012, paper no. 27.

[22] KIM, Y.K. ; KIM, S.I. ; RHYU, S.H. ET AL.: Tolerance Allocation in Electric Machine Based on Stochastic Response Surface Methodology. In: *Proceedings of the XIX International Conference on Electrical Machines.* Rome, Italy, 2010. S. 1-4.

[23] CHEN, M.-C.: Tolerance synthesis by neural learning and nonlinear programming. In: *International Journal of Production Economics* 1 (2001), S. 55-65.

[24] MOST, T. ; WILL, J.: Meta-model of optimized prognosis – An automatic approach for variable reduction and optimal meta-model selection. In: *Weimar Optimization and Stochastic Days 5.0*. Weimar, Germany, 2008.

[25] FORTINI, E.T.: *Dimensioning for Interchangeable Manufacture*. New York, USA : Industrial Press, 1967.

[26] STOCKINGER, A. ; WARTZACK, S.: Application of sampling methods for statistical tolerance analysis. In: *Weimar Optimization and Stochastic Days 7.0*. Weimar, Germany, 2010.

[27] STOCKINGER, A. ; WOKUSCH, A.-L. ; WALTER, M. ; WARTZACK, S.: Impact of probability distributions and covariances on numerical simulation results in engineering design. In: *Proceedings of the 16th International Conference on Industrial Engineering - Theory, Applications and Practice*. Stuttgart, Germany, September 2011, ISBN 97819346601-3-3.

[28] EFRON, B.: *The Jackknife, the Bootstrap, and Other Resampling Plans*. Philadelphia, USA : Society for Industrial and Applied Mathematics, 1982.