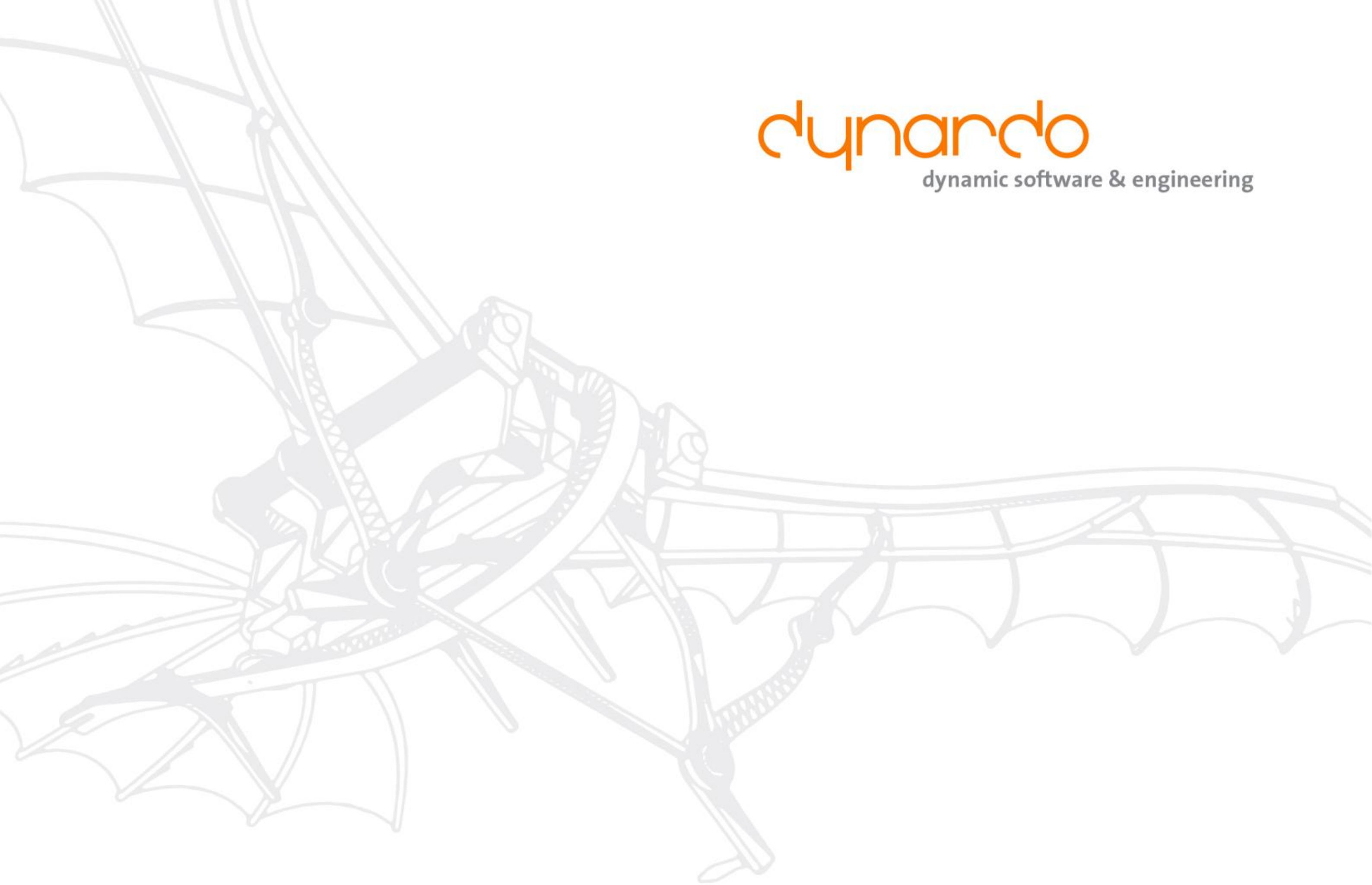

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Lectures

Recent Developments for Random Fields and Statistics on Structures

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Recent Developments for Random Fields and Statistics on Structures

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Abstract

This note presents new developments in random fields and statistics on finite element structures. It shows recent advances in computing the most relevant scatter shapes for large and very large structures and gives an outlook for the upcoming release of the software Statistics on Structures.

Keywords: random fields, statistics, finite elements, robustness analysis, SoS.

1 Introduction

Current developments in CAE often employ investigations on robustness, sensitivity or statistics of random effects on finite element structures. For example, an optimization must be accompanied by a robustness and reliability analysis. For this purpose, random influences are measured from experiments or are generated by Monte Carlo methods in conjunction with FEM simulation [1]. The performance of a structure is then assessed by statistical means [5]. Many processes involve random quantities being spatially distributed on the examined structure. Pointwise evaluation of the results, for example the search for maximum deformations or stresses, is possible using tools like optiSLang, but does not make use of the information inherent in the data and may even lead to misinterpretation if the localization is not tracked.

When utilizing random fields one is able to assess random effects as well as their localization. They provide several levels of insight: First, the distribution of scatter on the structure is observed and hot spots are located. Next, random field data can be decomposed into scatter shapes, which can be ranked by their contribution to the total scatter. These shapes can be used to reduce the number of random variables (which may be very large depending on the number of nodes and elements in the FEM mesh) and to reduce noise while only keeping the essential features of the random field. Further

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statistical analysis, mainly by means of Coefficients of Determination and Coefficients of Prognosis [7] allow for a ranking of the influence of random inputs on single scatter shapes.

DYNARDO developed the software *Statistics on Structures* (SoS) which is capable of decomposing random fields into scatter shapes, analyzing random properties on FEM structures, locating "hot spots" of variation and investigating correlations. SoS is mainly a post processor for statistics on FEM structures, i.e. for visualization of the descriptive statistics on the structure, visualization of correlations and CoD between random input and structural results, visualization of quality performance (QCS) and identification of spatial dependencies using random fields. Several successful applications are documented in the Dynardo online library at www.dynardo.de, for example [2,3,6]. [1] gives detailed insight into the abilities of the software, the theoretical background, the application in robustness and sensitivity analysis and the interpretation of the results.

The current software version SoS 2.3, however, takes a lot of computational resources when analyzing random fields. As a result, very large FEM meshes can not be treated. This limitation was the main motivation to rethink and redevelop the software core of SoS. This article presents the main ideas behind reduction of random fields and the new methodology in section 2. Section 3 illustrates first results using the new methodology. Further, section 4 presents the planned features of the upcoming SoS3 release.

2 Random fields

Let us consider a scalar field which is defined by a real-valued function value H over an n-dimensional space, i.e.

$$H : \mathbb{R} \to \mathbb{R}^n \; \mathbf{x} \to z, \quad z \in \mathbb{R}, \mathbf{x} \in \mathbb{R}^n$$

In most structural applications this function is defined in three-dimensional space, i.e.

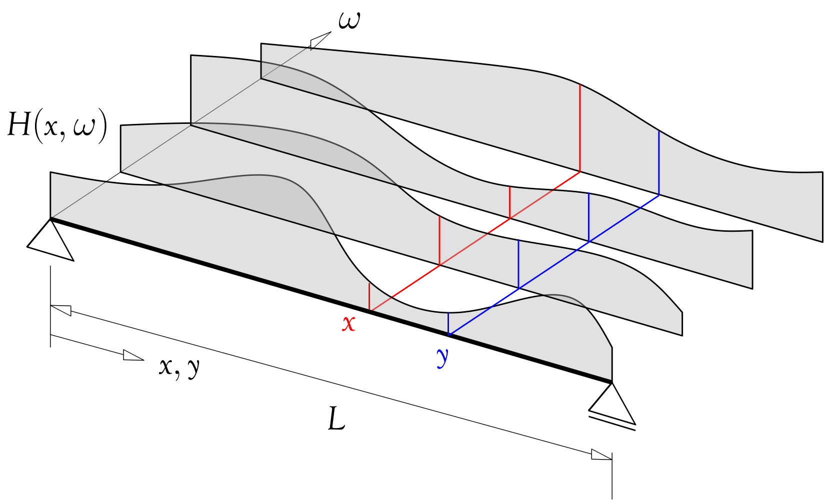

Figure 1: **Different realizations of a one-dimensional field.** Source: [6]

$n = 3$. Let us assume that the field given through H usually varies smoothly in space whereby points being very close to each other have similar function values. If the field is influenced by random quantities, then one may measure different realizations of the same field, see figure 1. Therein, four random samples ω of the same scalar field H defined in one dimension x are illustrated. Two points in space are highlighted in the figure, for which one can compute statistical measures as if they are individual random variables. For example, one can compute the mean value function

$$\bar{H}(\mathbf{x}) = \mathbf{E}[H(\mathbf{x})]$$

One can further compute correlation measures for the field values at different coordinates, for example by the covariance between two points $\mathbf{x}$ and $\mathbf{y}$

$$C_{HH}(\mathbf{x}, \mathbf{y}) = \mathbf{E}[\{H(\mathbf{x}) - \bar{H}(\mathbf{x})\}\{H(\mathbf{y}) - \bar{H}(\mathbf{y})\}]$$

with auto-covariance function C_{HH}.

The design space $\mathbf{x} \in \mathbb{R}^n$ is infinite large. In engineering applications one is interested in reducing the number of variables to a small finite number. A tool is to express the field H by a Fourier-type series expansion using deterministic basis functions ϕ_k and random coefficients c_k

$$H(\mathbf{x}) = \sum_{k=1}^{\infty} c_k \phi_k(\mathbf{x}), \quad c_k \in \mathbb{R},\, \phi_k \in \mathbb{R}$$

This transforms the field being originally expressed by the unknowns $\mathbf{x}$ to a space expressed by the unknowns $\mathbf{c}$. By truncating the series, a reduction of the number of variables can be achieved. The Karhunen-Love expansion states that an optimal choice of the basis functions is given by an eigenvalue ("spectral") decomposition of the auto-covariance function, i.e.

$$C_{HH} = \sum_{k=1}^{\infty} \lambda_k \phi_k(\mathbf{x}) \phi_k(\mathbf{y}), \qquad \int_{\mathbf{R}^n} C_{HH}(\mathbf{x}, \mathbf{y}) \phi_x(\mathbf{x}) d\mathbf{x} = \lambda_k \phi_k(\mathbf{y})$$

The obtained basis functions are orthogonal and the coefficients become uncorrelated.

When a scalar field is measured as a distribution on a FEM mesh (or on any other discrete space), the field H is represented by discrete values, i.e.

$$H_i = H(\mathbf{x}_i), \quad i = 1 \ldots N$$

In this case the spectral decomposition is given through

$$H_i = \sum_{k=1}^{N} \phi_k(\mathbf{x}_i) c_k = \sum_{k=1}^{N} \phi_{ik} c_k$$

or in matrix-vector notation

$$\mathbf{H} = \Phi \mathbf{c}$$

Again, a significant reduction in the number of variables can be achieved when truncating the series after a few items. The field H being measured in terms of a large number of values H_i (usually in terms of single values per node or finite element) is expressed through a small number of coefficients c_k. The "scatter shapes" ϕ_{ik} define the transformation basis.

By reducing the number of random variables, one improves the statistical significance for a small sample size (eliminates noise), reduces the numerical effort in statistical analysis and may simplify the representation of input/output relations based on meta models. The basis functions should be orthogonal reducing the computational effort for the projection (reduction) and its inverse transformation. As a side effect the random coefficients are uncorrelated simplifying the digital simulation of random fields.

Fast analysis of random fields

Statistics on Structures first computes the covariance matrix denoting the covariance between the individual discrete values H_i and H_k and then performs an eigenvalue decomposition to obtain the scatter shapes and random coefficients. By temporarily storing the covariance matrix, SoS is effectively limited to approximately 16,000 nodes/elements. In order to allow the analysis of larger structures, a mesh coarsening algorithm was introduced. Then one was able to handle FEM meshes up to 50,000 - 100,000 elements. The mesh coarsening is a user-controlled iterative procedure. Furthermore, depending on the problem a huge amount of variability may be lost. Local peaks of variability can be maintained only if they are in finely meshed areas (or where there is no substantial mesh coarsening).

With the upcoming version Statistics on Structures 3 a new algorithm will be introduced to perform the spectral decomposition: The idea is to eliminate the need to store the full covariance matrix. Hence, one avoids the storage requirements. The employed matrices are smaller and, as a side effect, the numerical algorithms for linear algebra perform significantly faster. The new strategy does not need any kind of coarsening algorithm. Thus, there are no accuracy losses due to coarsening, smoothening or interpolation. The algorithm allows a full analysis (i.e. the number of eigenvectors is equal to the number of random samples) in reasonable time. From that, an error estimation of the reduced order model is readily possible.

3 Numerical example

To demonstrate the new algorithm, the finite element mesh in figure 2 is considered. Using SoS2 the mesh was coarsened from 60836 elements and 33516 nodes to 21291 elements and 11215 nodes. For some quantities, the coarsening reduces the data variability significantly, for example for quantity 'plastic strain' to 59% of the original scatter. Table 1 lists the CPU times of both approaches.

Table 1: Numerical performance of Random Field Reduction

	number of nodes	nodes used in reduction	cpu time	peak memory
SoS 2.3	33516	11215	12h	
SoS 3	33516	33516	107s	0.8 GB

The 1st scatter shapes for the quantities "Thickness", "Thickness reduction" and "Plastic strain" are shown in figures 3, 4 and 5, respectively, for both algorithms. They agree very well. The same is true for the Coefficients of Prognosis (figure 6), which were

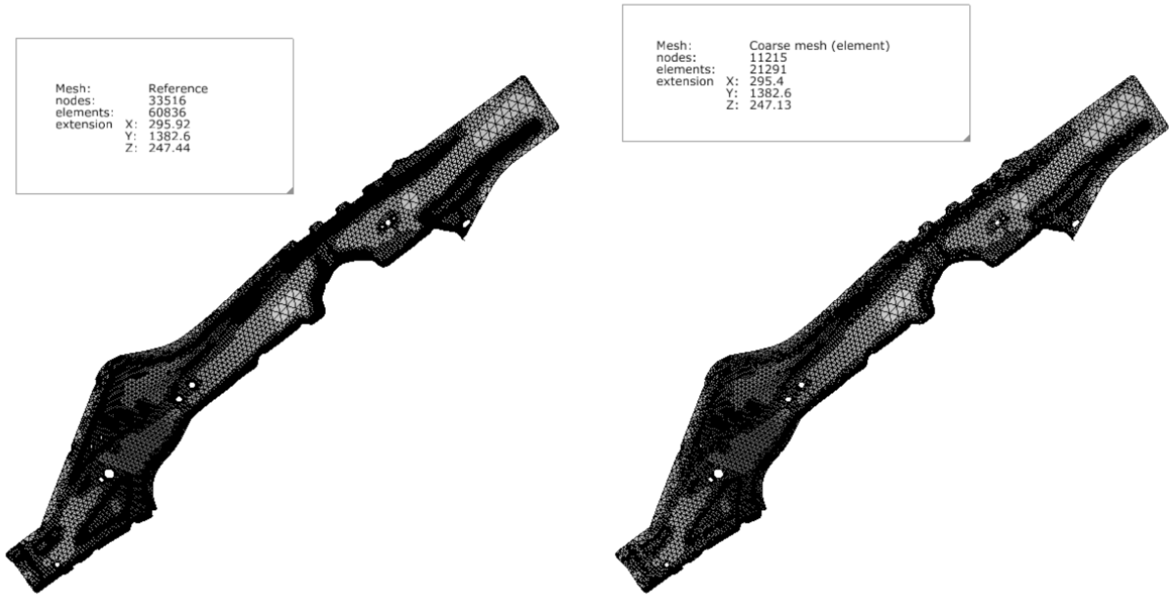

Figure 2: **Example mesh** Original mesh (left), coarsened mesh (right). Source: [4]

computed by optiSLang in a post-processing step and which describe the relation between the 1st variable in reduced space c_1 for the three quantities and the input variables.

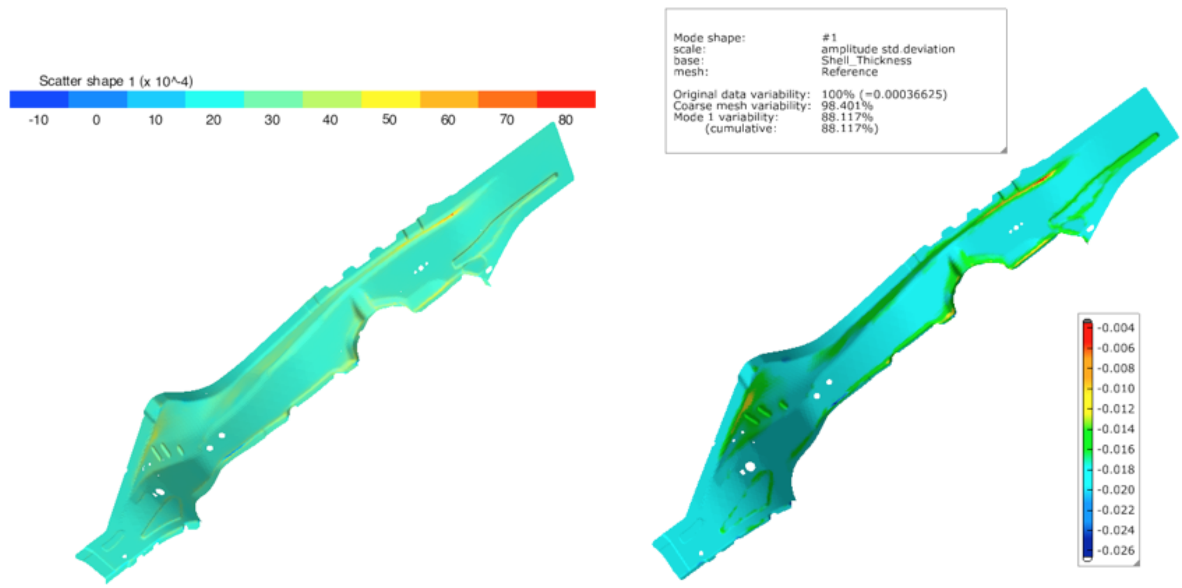

Figure 3: **Quantity 'Thickness' - 1st scatter shape.** Left: SoS 3, Right: SoS 2 [4].

Another example was analyzed using the new algorithms, see table 2. Using the new methodology, SoS 3 is possible to perform the full Random Field Reduction for a structure with 440,000 elements and 150 samples in short time.

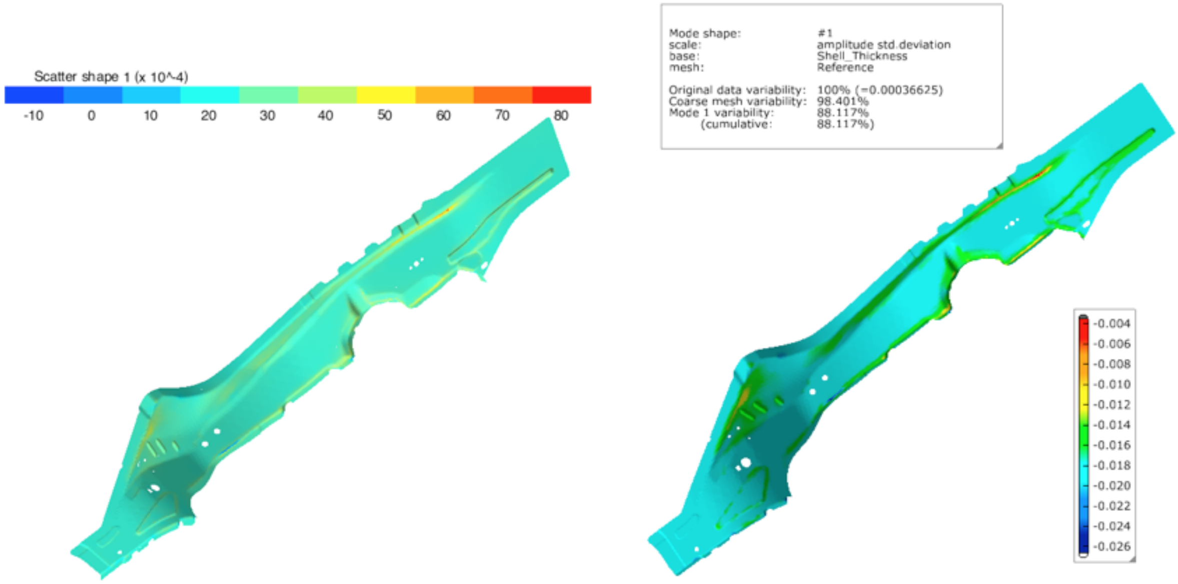

Figure 4: **Quantity 'Thickness reduction' - 1st scatter shape.** Left: SoS 3, Right: SoS 2 [4].

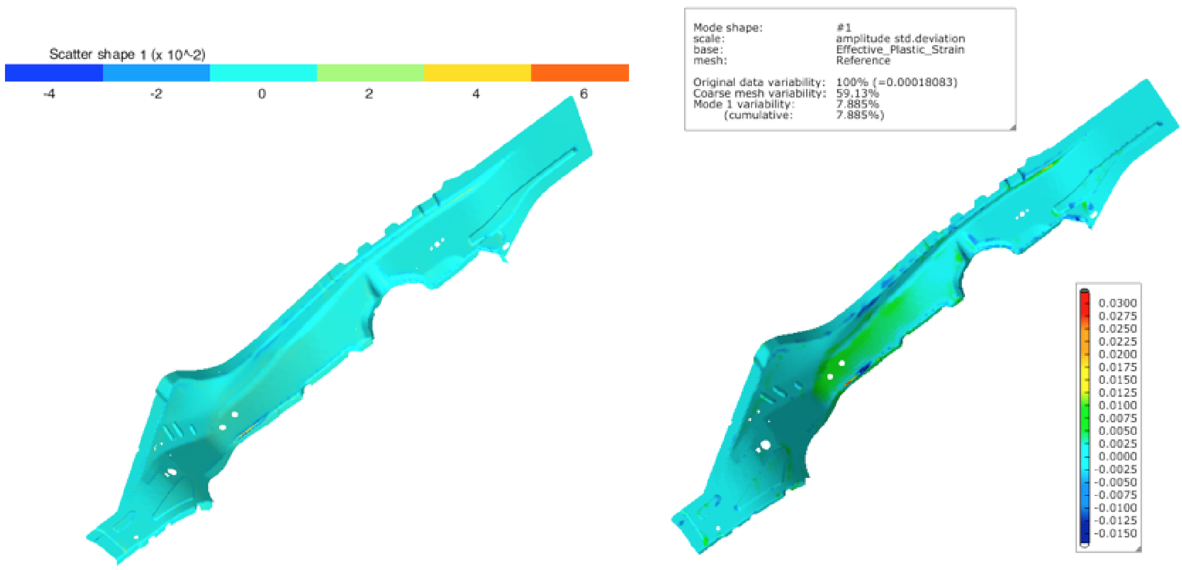

Figure 5: **Quantity 'Plastic strain' - 1st scatter shape.** Left: SoS 3, Right: SoS 2 [4].

Table 2: Numerical performance of Random Field Reduction for a single set of results

	number of elements	elements used in reduction	cpu time	peak memory
SoS 2.3	440,000	n.a.	n.a.	n.a.
SoS 3	440,000	440,000	510 s	5.9 GB

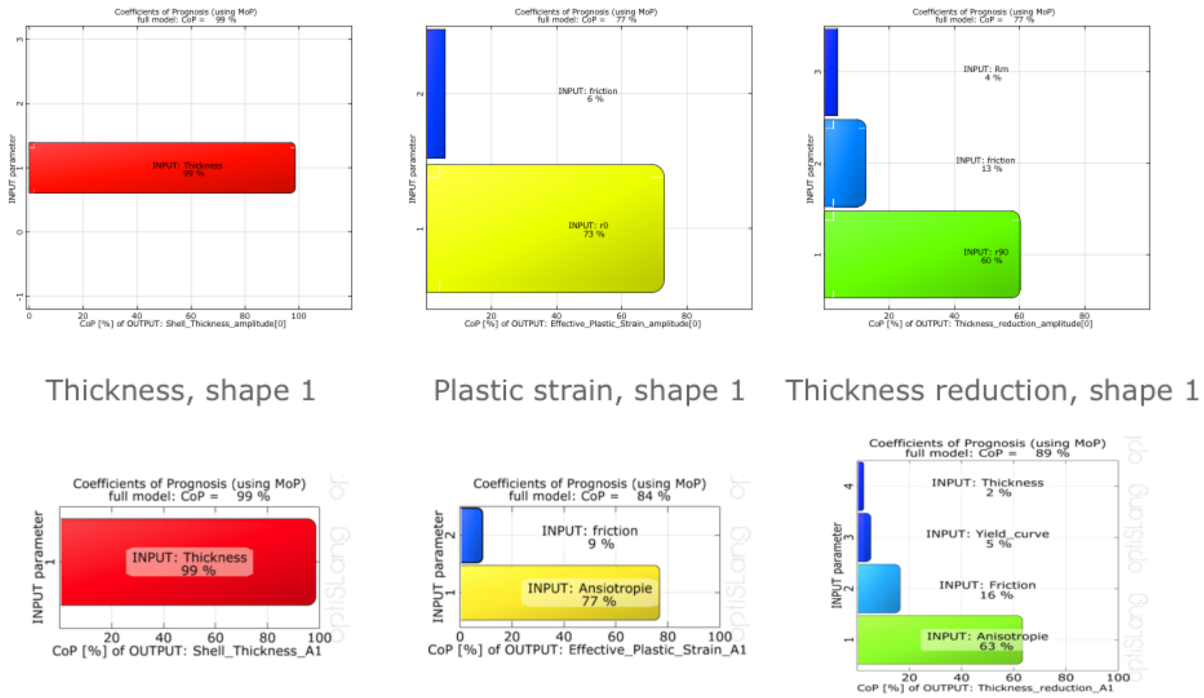

Figure 6: **Coefficient of Prognosis (CoP) for the 1st random coefficient and random input variables.**Top: SoS 2 [4] , Bottom: SoS 3.

4 Outlook: Statistics on Structures 3

The main goals of the software Statistics on Structures are

- to reduce the number of variables involved in the description of the fields.
- to identify input-output relations based on meta-models.
- to simulate realizations of random fields.

Using the new algorithms, we are able to achieve these within reasonable time. Furthermore, the new version is finally capable to treat large and very large finite element meshes. This allows us to extend the application of SoS to new areas. In detail, SoS 3 targets at:

- Estimation of second-order statistics (mean value function, covariance function) based on sampled data, either from measurements or from computations such as Monte Carlo studies.
- Reduction of random field models based on second order statistics by applying Karhunen-Love expansion.
- Digital simulation of random field samples.
- Import and export of random field samples from/to third party finite element codes as element/node properties (development primarily focuses on LS-Dyna shell meshes).

- Implementing a FEM core with support for 1st and 2nd order shell and continuum finite elements.
- Mapping of random field data between incompatible meshes (typically fine-coarse, but also dislocated meshes)
- Identification of geometrical deviations
- Correlation analysis between input/output variables (utilizing optiSLang).
- Identification of important input variables (utilizing optiSLang)
- Treat exceptional situations (e.g. eroded elements).

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