Sensitivity Analysis of a Turbo Compressor with Physical Parameterization

Johannes Einzinger
ANSYS Germany
Turbo Optimization Strategies with Meta-Model

Traditional Design/Optimization Strategy

- Primary Design (Empiric)
- Manual Improvement, CFD
- optiSLang – small Parameter Range

"Optimization does it all"

- Primary Design (Empiric)
- optiSLang – large Parameter Range

Works excellent, since Engineer + optiSLang = perfect Combination
Key: small Parameter Range
Draw back: needs to be done for each new Design

Strategy usually fails, bad Results
Problem: large Parameter Range
Benefit (if it works): investment upfront, fast Optimization for each new Design
Agenda

• Turbomachinery Design Principals
• Traditional Parameterization and Limitations
• Physical Parameters
• Sensitivity Analysis
Turbomachinery – Conservation Principals

Mass Conservation
\[ \sum \int \rho \vec{c} \vec{n} \, dA = 0 \]

Momentum Conservation
\[ \sum \int \left( \rho \vec{c} \vec{n} \right) \vec{c} \, dA = \sum \int (p \vec{n} + \vec{\tau}) \, dA \]

Energy Conservation – Euler Equation
\[ \Delta h_t = P_{Mech} + Q_{Therm} \quad \Delta h_t = \Delta (uc_u) \]

Total Enthalpy Gain = Work due Rotation

Kinematics – Velocity Triangle
\[ \vec{c} = \vec{w} + \vec{u} \]

Circumferential Velocity

Absolute Velocity

Relative Velocity

Thermodynamics, ideal Gas

Equation of State
\[ p = \rho RT \]

Caloric Equation
\[ h = c_p T \]

2nd Law
\[ dh = T \, ds + \frac{dp}{\rho} \]
Geometry Parameter

Traditional Turbomachinery Parameterization
• B-Spline for Hub and Shroud
• Line/Curve for Leading- and Trailing Edge
• Blade Angles on n Layers
• Thickness Distribution
• Boundary Conditions
Inducer Geometry

Mass Conservation
\[ \dot{m} = \rho A c_m = \rho r_1^2 (1 - v^2)c_m = \text{const} \]

Parameter Modification

Increased Speed of Revolution \( \Omega \)

Reduced Inducer Radius \( r_1 \)

Flow Separation, less efficient

Large Parameter Variations leads to bad Velocity Triangles

Overcome: Constraints...
"Hidden" Impact of Geometry Change

In general: Geometry Change will "shift" Operating Point! Operating Point Change has similar Impact like Geometry Change!

Part Load

Best Efficiency Point

Over Load

Large Parameter Variations leads to different Operating Points, how to compare? Overcome: Constraints...
Discharge Geometry

Ideal Design Configuration

Swirl in Direction of Rotation

Reduced Channel Height $b_2$: counter Swirl “Turbine Mode”

Increased Impeller Radius $r_2$:  
- low $c_m$  
- high $u_c$  
- high $dp/dr$  
- Flow Separation

Mass Conservation

$$\dot{m} = 2\pi r_2 b_2 \rho c_m = const$$

Large Parameter Variations leads to bad Velocity Triangles  
Overcome: Constraints...

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Dimensionless Analysis

• Ambient Conditions $\rightarrow$ Reference Speed of Sound $a_t$
• Impeller Tip Speed $\rightarrow$ Reference Velocity $u_2$
• Dimensionless Velocity Triangle and Euler Equation

\[
\frac{h_{t2}}{h_{t1}} = 1 + (\kappa - 1) \frac{\frac{u_2^2}{\kappa RT_{t1}}}{\frac{M_{t,u}^2}{u_2}} \frac{c_u}{\psi}
\]

For all: meaningful upper and lower Bounds possible, without Constraint.

$\vec{c} = \vec{w} + \vec{u}$

$\psi$: Load Coefficient

$\beta_2$: Blade Angle

$\varphi = \frac{c_m}{u_2}$: Flow Coefficient
Inducer Design

Native Parameters: \( c_1, w_1, u_1, \beta_1, r_1, \Omega, T_{t1} \)

Dimensionless: Velocity Triangle + Mass Flow Definition

Design Requirement: low \( M_{w1} \rightarrow \) low Friction

\( \beta_1: \text{Blade Angle} \)

\( \beta_1 = -45^\circ \)

\( \beta_1 = -65^\circ \)

\[ \Theta = \frac{m\Omega^2}{Q_t a_t^3} \]
Discharge Design

Native Parameters: $c_2, \ w_2, \ u_2, \ \beta_2, \ r, \ \Omega, \ T_{t1}, T_{t2}, \ p_{t1}, p_{t2}$

Dimensionless: Velocity Triangle + Euler Equation

Additional Correlation: $\Pi = f(\Theta, n_s)$

Pressure Ratio $\Pi$

$\beta_2 = 0^\circ$

$\beta_2 = -75^\circ$
Workflow

WB-Python Script to compute Main Dimensions wrt to Input Parameter

“Secondary” Geometry Parameter
Parameterization respects Cordier Correlation Polytropic Efficiency in Meta-Model Sampling between 89.1% and 95.5%
Result – State Variables

Polynomial Efficiency = \( \frac{\kappa - 1}{\kappa} \frac{\ln(\text{Total Pressure Ratio})}{\ln(\text{Total Temperature Ratio})} \)

CoP=91%

CoP=96%

CoP=97%
Results – Design Correlations

Channel Height $b_2/r_2$

Inducer Radius $r_1/r_2$

Blade Length $L/r_2$

Useful Information for further primary Design

- Channel Height: $b_2/r_2$, CoP=99%
- Inducer Radius: $r_1/r_2$, CoP=99%
- Blade Length: $L/r_2$, CoP=97%
Summary – Parametrization

- Geometry Parameters $G_i$ are very easy to define and vary
- Geometric and/or Physical Constraints might be needed → Use Relative Parameters $S_i$
- Geometric and/or Physical Constraints can become complex → Use Relative Parameters $P_i$

Relative Parameter avoid constraint Equations

Physical Parameter transforms complex Shape to Square

no Constraints, no Problem
Key to Successful Optimization

- Parameterization
- Simulation Workflow
- Optimization Tool

optiSLang