Meta-Model Method (ROM) to compute the Bearing Coefficients for Rotor Dynamic Applications

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Agenda

Application Areas of Fluid Film Bearings

Modelling
3D Navier-Stokes (CFD)
Reynolds Equation (Mechanical)

Reduced Order Model
Application Areas of Fluid Film Bearings

• Why Fluid Film Bearings?
  • Simple Construction
  • Good Damping Characteristics
  • High Load and Speed
  • High Precision Applications

• Critical to Machines overall Reliability!!
Modelling – Overview

Accuracy and Usability

Efficient Coupling with Rotor Dynamics

How to compute the Coefficients???

Simplified Navier-Stokes Equation

Limited Modelling

Reynolds Equation

Spring & Damper

Computational Effort

Thermal and Mechanical Deformation

CFD and ROM for Mechanics, Rigid Body

Non-Linear Fluid-Structure Interaction

CFD and ROM

3D CFD

Full Navier-Stokes Equation

\[
\begin{align*}
\begin{bmatrix}
C_{xx} & C_{xy} \\
C_{yx} & C_{yy}
\end{bmatrix}
\begin{bmatrix}
\dot{u}_x \\
\dot{u}_y
\end{bmatrix}
+ 
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y
\end{bmatrix}
= 
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix}
\end{align*}
\]

Simplified Navier-Stokes Equation

\[
\frac{\partial}{\partial x} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \frac{U}{2} \frac{\partial h}{\partial x} + \frac{\partial h}{\partial t}
\]
Simulation Procedure

• Stiffness and Damping is wrt to Equilibrium Position:

• **Calculate Equilibrium Position**

• **Stiffness Coefficient:**

\[
\begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\
\frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2}
\end{bmatrix}
\]

Repeat Simulation with varied Position \( \rightarrow K = \frac{\Delta F}{\Delta x} \)

• **Damping Coefficient:**

\[
\begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\
\frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2}
\end{bmatrix}
\]

Repeat Simulation with varied Velocity \( \rightarrow C = \frac{\Delta F}{\Delta x} \)

2-Way-Coupled Fluid-Structure Interaction!
High Computational Effort!
Reduced Order Model (ROM) Approach

- Design of Experiments
- Variation of Eccentricity, Attitude Angle, ...
- Measure Reaction Force

- Response Surface is calculated (=ROM)
  - Reaction Force = \( f(\text{Eccentricity, Attitude Angle, ...}) \)

- Optimization to find
  - Eccentricity, Attitude Angle, ...
  - For given External Force

- Stiffness is Derivative of Response Surface
- Damping is calculated at Equilibrium
3D Navier-Stokes in CFX

- 3D Resolution & Kinematics
  - Steady Simulation
    - Re-Meshing for each Position
    - Mesh Morphing
  - Transient Simulation
    - Mesh Morphing
- → Analytical Mesh Morphing

Outlook: Analytical Mesh Morphing

\[
\Delta \bar{x} = r_{rel} \cdot \Delta \bar{x}_{Shaft}
\]

\[
r_{rel} = \frac{r - r_i}{r_a - r_i}
\]
Equilibrium

- **Steady State: Design of Experiments**
- Variation of Eccentricity, Attitude Angle, ...
- Measure Reaction Force
- Get Equilibrium from Response Surface

- **2-Way-Coupled FSI with Rigid Body**
  - Rigid Body Dynamics is solved in CFX!

\[ m \cdot \ddot{x} + d \cdot \dot{x} = F_{\text{Fluid}} - m \cdot g \]

Artificial Damping to avoid overshoots, zero for Equilibrium!
Result: 2-Way-Coupled FSI with Rigid Body

Position, constant for increasing Time → Steady equilibrium

Orbit Plot
Position in X-Y Plane

Force, constant for increasing Time → Steady equilibrium

Position y
Position x

Force Y = Load

Force X = 0
Calculate Stiffness and Damping

• Calculate Equilibrium Position

• Stiffness Coefficient:
  \[ \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} \end{bmatrix} \]
  Repeat Steady Simulation with varied Position \( \Rightarrow K = \frac{\Delta F}{\Delta x} \)

• Damping Coefficient:
  • Transient Simulation with oscillating Shaft (one for x and one for y) until periodic Result
  • Dissipation Work: integrate Force x/y and Velocity x/y over one Period
  • Normalization \( \Rightarrow \) Damping

\[ W_{\text{Diss},ij} = \int_{0}^{T} F_{j} \cdot \dot{x}_i \cdot dt \]
\[ C_{ij} = \frac{W_{\text{Diss},ij}}{\omega \cdot x_i^2} \]

\[ x_j(t) = x_0 \cdot \sin(\omega t) \]
Bearing Model – Mechanical

Definition of Bearing Elements by Generic Snippet

Input Parameter:
- Eccentricity
- Attitude Angle
- Velocity X
- Velocity Y
- Speed of Revolution

Output Parameter:

<table>
<thead>
<tr>
<th>Results</th>
<th>my_fyX</th>
<th>my_fyY</th>
<th>my_SoX</th>
<th>my_SoY</th>
</tr>
</thead>
<tbody>
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<td>4935.5</td>
<td>0.43692</td>
<td>0.3036</td>
</tr>
</tbody>
</table>
Calculate Stiffness and Damping

Vary Equilibrium Position with $\Delta x$, get Reaction Force and calculate Stiffness:

$$K_{ij} = \frac{F_j(x_{eq} \pm \Delta x_j) - F_{j,eq}}{\Delta x_i}$$

Shaft Velocity can be applied in steady State Simulation!
Calculate Reaction Force and Damping

$$C_{ij} = \frac{F_j(x_{eq}, \Delta \dot{x}_j) - F_{j,eq}}{\Delta \dot{x}_i}$$

Perturbation and Induced Forces

$$\frac{\partial}{\partial x} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12\mu} \frac{\partial p}{\partial z} \right) = \frac{U}{2} \frac{\partial h}{\partial x} \left( \frac{\partial h}{\partial t} \right)$$

$\frac{\partial h}{\partial t} \sim \Delta \dot{x}_i$
Reduced Order Model / Response Surface

**Input:**
- Eccentricity
- Attitude Angle
- Rotational Speed
- Equilibrium Force (see later)

**Output:**
- Force X,Y
- Sommerfeld-Number
- Equilibrium Objective (see later)

Reaction Force = \( f(\text{Eccentricity, Attitude Angle, ...}) \)
Reaction Forces X

Equilibrium: Reaction Force X = 0

Reaction Force X depends on Eccentricity, Attitude Angle and Rotational Speed

Equilibrium

\[ n = 2500 \text{ rpm} \]

Equilibrium

\[ n = 4500 \text{ rpm} \]
Reaction Force Y

Equilibrium Force = Reaction Force Y

Reaction Force Y depends on Eccentricity, Attitude Angle and Rotational Speed

Attitude Angle
Rotational Speed n
Eccentricity

Attitude Angle=–45°
Attitude Angle=–60°

Equilibrium 15 kN
Sommerfeld Number

\[ S_0 = \frac{p_m \cdot \Psi^2}{\eta \cdot \omega} \sim \frac{F}{\Omega} \]

Number of relevant Parameter reduced by non-dimensional Analysis

\( S_{0x} \) shows Line for all Equilibrium Positions

\( S_{0y} \) shows Equilibrium on Axis for non-dimensional Load

Objective \( \sim \frac{1}{\Omega} \sqrt{F_x^2 + (F_y - F_{\text{Load}})^2} \)

Equilibrium \( S_{0y} = S_0(F - \text{Load}, n) \)

Equilibrium \( S_{0x} = 0 \)
Find Equilibrium Position

Input:
- Eccentricity = ?
- Attitude Angle = ?
- Rotational Speed = 5000 rpm
- Equilibrium Force = 7000 N

Optimization:
- Objective → 0
- → Equilibrium

Optimum/Equilibrium is validated by Simulation!!!

Relative Difference 0.5%
CoP = 98%

Reaction Force = f(Eccentricity, Attitude Angle, ...)

Optimization on Response Surface → Equilibrium
optiSLang Usability

**Push-Button Import of Meta-Model**  
**Additional Calculation in Excel with MOP-Solver**
Summary

Calculate Equilibrium: Reduced Order Model is much more efficient than 2-Way FSI !!!

- Design of Experiments
- Variation of Eccentricity, Attitude Angle, ...
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