Statistical Analysis of a mistuned Compressor

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Outline

What is Mistuning?

Turbo Machinery, FSI-Coupling

Mistuning, Modelling

Mistuning, Applications
What is Mistuning?

- Why does Blade x break?
  - Local Production Error?
  - Local Material Error?
  - Local Overload?
  - Local Erosion?
  - ...

- Non cyclic System due to
  - Allowed Production Tolerances
  - Small Erosion

- Mistuned System

Rotor Damage at Blade x

CAD-Model
(=Tuned System)

Real-Model
(=Mistuned)
Transient CFD – Dynamic Loading

\[ f(t) = \sum_{n=0}^{N} a_n \cdot \cos(n \cdot \omega \cdot t) + b_n \cdot \sin(n \cdot \omega \cdot t) = F(\omega) \]

Fourier Transformation

\[ P_{\text{max}} = \begin{cases} 0.1 \text{ [MPa]} & \text{for } n=0-3 \\ 5000 \text{ [Pa]} & \text{for } n=4-7 \end{cases} \]
Pre-Stressed Modal Analysis

Eigen Frequencies:

Eigen Modes $\Phi$:

Model Order Reduction:

$$u = \Phi_1 \cdot q_1 + \Phi_2 \cdot q_2 + \Phi_3 \cdot q_3 + \ldots$$
Dynamic Mechanical Analysis

Fluid Pressure Excitation Order

Critical Mode

Modal Force

Modal Coordinates @ Critical Frequency

Critical Mode Amplitude = f(Ω)
Mistuning, Modelling

1 DOF Minimal Modell

2 Sector Modell with Point Mass

CMS Modell (1 Sector!)

Full Model with Random Fields
1-DOF Minimal Model

- Each Blade Oscillation is represented by one mode, i.e. DOF
- Parameter: cantilevered blade frequency and coupling stiffness

- **Pro:**
  - efficient due to reduced order model
- **Con:**
  - limited in modelling capacities
2 Sector Model with Point Mass

- Cyclic System with 2 sectors
- **Additional Point Mass on 2\textsuperscript{nd} Sector → Mistuning**

- **Pro:**
  - easy to apply
- **Con:**
  - regular pattern implicit included → detuned
CMS Model

- Cyclic Model + non-cyclic Mistuning
  - Proportional Mistuning
  - Intended Mistuning
  - ...

- Different Stress Levels on Sectors/Blades

  - Pro:
    - efficient: single sector mesh required

  - Con:
    - small Mistuning
Full Model with Random Fields

• Measured / Assumed Variation $\rightarrow$ Random Fields (Mode Shapes of Correlation Matrix)
• Random Field is morphed on tuned System Mesh

• Pro:
  – Model for large Mistuning
  – efficient handling of statistics

• Con:
  – computational effort
Application & Best-Practice

• Reason for small Coefficient of Prognosis:
  – Number Design Points
  – Numerical Error
  – Model Error
  – Multiple Mechanism

• Number of Design Points for Meta-Model depends on:
  – Number of important parameters
  – Nonlinearity of Response Surface

Objective for Meta-Model: Maximal Coefficient of Prognosis
1-DOF Minimal Model

• 8 Sector Model
  – 16 Input Parameters
  – Normal Distribution
  – CoV=1%

• Simulation:
  – Modal Analysis
  – Forced Response, Excitation @ 1 Sector

• Output:
  – 5 Eigen Frequencies
  – Frequency Sweep:
    • Signals @ Sectors=f(Ω)
    • Signals for Nodal Diameter @ Eigen Frequency

CoP is increasing with Number of Design Points!

<table>
<thead>
<tr>
<th>CoP [%]</th>
<th>N=200</th>
<th>400</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq. ND=0</td>
<td>96.6</td>
<td>98.7</td>
<td>99.1</td>
</tr>
<tr>
<td>Freq. ND=1</td>
<td>75.7</td>
<td>89.5</td>
<td>94.5</td>
</tr>
<tr>
<td>Freq. ND=2</td>
<td>81.4</td>
<td>92.6</td>
<td>94.0</td>
</tr>
<tr>
<td>Freq. ND=3</td>
<td>79.6</td>
<td>94.2</td>
<td>95.0</td>
</tr>
<tr>
<td>Freq. ND=4</td>
<td>88.6</td>
<td>97.6</td>
<td>98.4</td>
</tr>
</tbody>
</table>

\[ F(t) = F e^{j\Omega t} \]
1-DOF Minimal Model

Eigen Frequency Shift!

Tuned System

Extrem Response

Misuned System

Normalized Response @ Sector 1

Excitation Frequency

Lower Response at Resonance

Significant higher Response at Resonance
1-DOF Minimal Model

- **Significant Influence of Mistuning:**
  - Coefficient of Variation (CoV) = 1%
  - Response 70% higher!!

- **Extreme Response**
  - Misused System
  - Tuned System
2 Sector Model with Point Mass

- Radial Compressor with 2x8 Sectors
- 10 Point Masses on even Sector
- Weibull Distribution, CoV 1% Sector Mass
- Excitation with CFD Loads
- Excellent CoP for all Output Variables with 100 Design Points
2 Sector Model with Point Mass

- Disp. Variation ~1% for Blade 1 and 2
- All Input Parameters are important wrt to Output

Blade 1, CoP=98%
Blade 2, CoP=99%
Blade 1
Blade 2
Maximum Displacement
Summary and Outlook

• Summary
  – Mistuning has significant Influence to Bladed-Rotors (and other structures)
  – Efficient Mistuning Models are available

• Outlook
  – Application of CMS Model, → Release 16
  – Application of Random Fields

Use Simulation and Statistical Analysis...

...to avoid: