Lectures

The Calibration of Measurement and Simulation as Optimization Problem

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The Calibration of Measurement and Simulation as Optimization Problem

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Summary:
The calibration of measurement and computation is one of the classic problems of model validation. If the difference between measured and computed data is too large, an optimization problem for minimizing the difference can be formulated. Optimization problems of calibration of measurement and computation are often also called identification problems. In the past this problem was normally solved via iterative calibration of single variables. Through the availability of parameter optimization programs the iterative calibration can be automated and more complex problems can be handled. Such automated identification procedures become more and more important due to today's speed of innovation in product development. Within virtual prototyping virtual testing is the key to reduction of hardware tests. Only if calibration of measurement and simulation is successful in reference experiments then one can assume that all of the phenomena relevant for the real testing results are included in the virtual model and therefore models that produce competent results of prognosis are available for virtual testing.

In this paper a short overview of methods of sensitivity studies and optimization strategies is given. Their applicability for as automated identification as possible is then discussed using practical examples.

Keywords:
system identification, optimization, sensitivity studies, statistic methods
1 Introduction

The comparison between results of measurements and corresponding result values of the numerical model will always stand at the beginning of the calibration. If the difference is too large, then the search for a better calibration can be formulated as an optimization problem. The “optimization variables” are the variables of the numerical model in which the differences between measurement and computation are assumed and respectively those whose values are unknown or uncertain. Then a calibration between simulation and experimental result in a design space, which is defined by varying “input variables” of the numerical models, is computed using optimization routines. Such a conceptual formulation demands specific requirements from the optimization method, as discussed in the first part of this paper.

The complexity of such a problem varies from calibration of a single variable to the identification of unknown system properties. Whereas the calibration of single result variables by “adjusting” single variables is a process which every designer is probably familiar with, identification problems of unknown variables and the corresponding system properties can become very complex.

In addition the calibration of single values loses significance when dealing with heavily scattering results of measurements. If one has to assume, that the scatter of results of measurements also has to be expected in reality, the question comes up, which of the possible results of measurements is to be identified. A first step in order to consider scatter of result variables is the averaging of the scattering variables using the mean value theorem. Then at least is made sure that the calibration of the expectation (mean value) is realized and that at average the prognosis ability is given.

A calibration with the mean values of the measurements is insufficient if prognosis ability of the simulation is to be insured beyond the expected interval of scatter. Then validations should also be made at the considered interval borders.

The observed scatter of the result variables naturally includes important information, beyond the problem of the ability of prognosis of all possible results, about the scatter which is to be expected in reality. Such details about the input scatter to expect which leads to the observed output scatter are necessary for example for robustness evaluations in virtual product design [9]. The problem is then broadened by the identification of statistical properties of the input variables which are connected with the resulting scatter. Therefore strictly speaking statistical measures of the scattering input and output variables have to be identified. If the ability of prognosis of an area of scatter is to be secured, the identification of the associated areas of input scatter that are responsible for the output scatter would be sufficient. If however statements about the probabilities of the transgression of limits of the result variables are of interest then the distribution functions of the input variables, which lead to the observed output scatter, have to be identified. In the extreme case this leads to the identification of n-variable-sets to n-experiment-results. The histograms of the n-identified variables of each set form the base of the distribution function.

From our practical experience the key to the success of an identification often lies in the definition of the design space as well as the objective function in addition to efficient optimization algorithms. A promising optimization problem often can only be formulated if the design space can be assembled from sensitive parameters. Therefore it is recommended to perform a sensitivity analysis of the potential design space and to constrict the variable set to the sensitive parameters of significant input variables before optimization. It shall be pointed out that sometimes when solving identification problems the phenomena arises that with increasing number of optimization variables the quality of calibration only increases at first view. The optimizer is able to minimize the objective function more easily by having more degrees of freedom but also provides a number of variables which obtain random values. A translation of the variable set to plausible physical variable and respectively the “filtering” of identified variables and random or “misaligned” variables is not trivial. Therefore a 90% calibration with a few variables which are identified as sensible physical variables is better than a 95% calibration with a multitude of random or misaligned parameters.

At the same time within the sensitivity analysis it should be tested if the result which is to be calibrated is situated in the design space of the virtual model. If it can be assumed with a high probability, that the measurements which are to be calibrated lie beyond the variation space of the sensitivity study then the reasons should be discussed. Either further differences between measurement and simulation can be identified and obtained as “optimization variable” or the numerical model does not include the underlying functional mechanism or the measurements are afflicted with errors.
2 Sensitivity Studies

Sensitivity studies are recommended in order to verify, if the design space was chosen from “sensitive” parameters to identify and if a calibration with results of measurement seems realistic. Parameter studies, that is the variation of single parameters, belong to the everyday life of an engineer for a long time now. In analogy thereto design of experiment methods, which systematically calculate single parameters and combinations of parameters, can be used in small parameter spaces. If the dimension or the nonlinearity of the parameter space increases, stochastical sampling strategies are to be favoured for creating supporting point sets.

A further advantage of stochastical sampling strategies compared to design of experiments is, that they furthermore permit a statistical evaluation of sensitivities via correlation hypothesis (which optimization variables operate on which result variable and how) an variation analysis (estimate the possible variations of the result to align in the chosen design space). Most important statistical variable for sensitivity of the optimization variables on significant result variables is the measure of determination. In figure 1 for example the measure of determination shows, that 95% of the variation of the maximum force in section 35 results from linear correlation and the “most sensitive” input variables are the yield stress as well as the thickness of two sheet blanks (Part 1007 and 1009 ). Consequently the sub space of identification of this result variable can be reduced to the four sensitive parameters. From the histogram one can read off, that the result variables in the design space varies at least between 69515 N and 85756 N. If the measurement result to identify lies in this variation space one can assume with high probability that a calibration in the subspace of the four sensitive variables is possible. For description of further statistical measures [10] shall be referred.

Fig. 1 Measures of determination and histogram of a single result variable

Therefore sensitivity studies enable a reduction of the parameter space for subsequent optimization problems. The previous knowledge obtained from the sensitivity studies about properties of the design space in addition is very helpful for an adequate formulation of constraints and objective function. From the computation of the sensitivity studies adequate starting points for gradient optimization, adequate starting approximation spaces for adaptive response surface methods or input information for starting generations of evolutionary search strategies can be obtained.

3 Solving of the Identification Problem using Methods of Optimization

Basically at least three categories of algorithms are available for solving the optimization problem: mathematical methods of optimization using gradients (gradient method), response surface methods (RSN) and stochastical search strategies.

3.1 Mathematical Optimization Methods using Gradient Information

Mathematical optimization methods [7], which determine the search direction using gradient information, offer the best convergence behaviour of the above mentioned methods. But they also have the greatest requirements on the mathematical composition of the numerical problem formulation, on continuity, differentiability, smoothness, scalability as well as the accuracy of the gradient determination.

Most critical from a practical point of view is the unavailability of analytical or semi-analytical gradients adverse important result variables to estimate and respectively the impracticability of numerical gradients for example when dealing with heavily noise afflicted problems, non differentiable problems
or problems of accuracy when determining numerical gradients. Successful practical application is consequently concentrated on optimization problems with continuous optimization variables with mathematically adequate problem formulations where suited gradients can be calculated. Ideally gradient methods should start close to the optimum. Therefore gradient methods are often used in order to verify if the pre-optimized parameter set can be further optimized.

### 3.2 Response Surface Methods

If the amount of optimization variables is limited to a few variables (5 to 15) then response surface methods [4] offer attractive possibilities of optimization. This method create an approximation of the design space using an approximation function on a suitable set of supporting points (samples of the variable space). The support points thereby should be determined using optimal support point pattern (Design of Experiments –DOE) for the approximation function. The approximation function usually has smooth mathematical properties that for the search for the optimum in the subspace mathematical methods of optimization can be used. Weak point of the response surface methods is the proof that the approximation at points of interest in the design space is sufficient and respectively accurate enough for the optimization. To secure the approximation quality adaptation Response Surface schemes are used. Hereby adaptive response surface methods (ARSM) which zoom and scroll the approximation space until the optimum converges on the response surface are the most successful [8]. The critical value from practical view first of all is the number of optimization variables. Therefore response surface methods are used in small dimension of the most sensitive optimization variables which have been determined before using sensitivity studies. Designs which have been pre-optimized in such a manner can be used as starting point for gradient optimization or as input information of evolutionary search strategies.

### 3.3 Evolutionary Search strategies

If the aforementioned algorithms do not lead to the desired goal stochastic search methods, of which the evolutionary algorithms with the subdivisions genetic algorithms [1] and evolutionary strategies [6] are the most successful, remain for solving the problem. The term stochastic search method is used as “random” events lead to the change in design. Important differentiating factor between genetic algorithms and evolutionary strategies is the method of evolutionary development of the optimization variables. Most important evolutionary process of the genetic algorithms is the random substitution of genes (optimization variables) between two parent designs to produce a descendant. Most important evolutionary process of evolutionary search strategies is mutation (random change) of single genes of a parental design to produce a descendant. Genetic algorithms thereby are especially useful for a relatively wide-ranging search in the design space. Therefore they are often used as a “global” search of a possible calibration. Evolutionary strategies are especially useful if a proper previous knowledge is available in the starting generation. Starting with pre-optimized designs from genetic search strategies or ARSM runs evolutionary strategies can be used for local optimization for fine-tuning. Depending on the settings of the replacement and mutation operators hybrids between genetic and evolutionary search strategies can be presented and used for combined global and local optimization.

### 4 Applications

In the following the potential application area of optimization methods for the calibration of measurement and computation shall be shown using practical examples. In all applications the software OptiSLang [5] was used for the optimization.


Bell towers of historic churches are a good example for structures whose dynamic behaviour is unknown. Most important dynamic animation of the structures is the ringing of the bell. At the Saint Michael church in Jena the stability of the bell tower under the loading case “ringing of the bell” was researched using dynamic measurements. Furthermore the reconstruction of the bell tower was accompanied by measurements and computations and it was tested if the reconstruction measures contribute to the desired strengthening of the bell tower.
The bell tower was dynamically animated using servo-hydraulic vibration generators. The velocity signals were recorded on different planes and from the calibration between transfer function of the measurement results and the numerical model eigenfrequency and eigenmode were identified.

For the calibration of the bell tower before reconstruction 39 mechanical properties (elasticity modules, raw densities, Poisson ratios, rigidity of connections/connection mediums and modal damping coefficients) were varied. The objective function was defined by differences in the eigenvalues and eigenmode (over a MAC-calibration). As optimization routine a genetic optimization algorithm was used. Different combinations of the objective function definition were used which all lead to similar calibration levels (fig. 4). In all variants of the objective function the frequency of the second mode could not be calibrated well. Near the third to fifth mode the calibration is moderately successful but depends strongly on the objective function, which leads to the conclusion that their weight in the objective function is relatively small compared to the first and second.

**The finite element model**

- 784 elements (mainly 8-node brick elements)
- 1458 nodes with 4337 active DOF
- approximation of vaults by plain slabs
- 12 groups of elements
- influence of church house modelled by spring elements

Fig. 2. Left – Bell tower during reconstruction, Middle – building history, Right – Positions of the vibration generator and the positions of the accelerometer.

Fig. 3 Finite element model
Model updating - structure before retrofit

- natural frequencies [Hz]

<table>
<thead>
<tr>
<th>Mode</th>
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<th>result - criterion 1</th>
<th>result - criterion 2</th>
<th>result - criterion 3</th>
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<td>1.449</td>
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<td>5.074</td>
<td>5.058</td>
<td>4.926</td>
</tr>
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</table>

- frequency response functions

Fig. 4 Calibration not yet reconstructed bell tower

For the calibration of the reconstructed tower 60 mechanical properties were varied. The objective function was defined by using differences between eigenvalues and eigenmode. In the first case (model 1) only the first and second eigenmode were calibrated, in the second case (model 2) the first 5 eigenmodes were calibrated. It could be shown, that a calibration of eigenfrequency and eigenmodes 1 to 4 is possible, but that in the defined design space conflicts between exist (Fig. 5). A satisfying calibration of all 4 first eigenmodes with one identified variable set could not be attained.

Model updating - structure after retrofit

- comparison of natural frequencies and mode shapes

<table>
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<th>description</th>
<th>nat. freq. model 1 [Hz]</th>
<th>description</th>
<th>nat. freq. model 2 [Hz]</th>
<th>description</th>
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<td>1.512</td>
<td>1st bending mode in SE-NW direction</td>
<td>1.674</td>
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<td>1.588</td>
<td>1st bending mode in SW-NE direction</td>
<td>1.562</td>
<td>1st bending mode in SW-NE direction</td>
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<td>4.78</td>
<td>2nd bending mode in N-S direction &amp; torsion</td>
<td>3.931</td>
<td>torsion</td>
<td>4.790</td>
<td>2nd bending mode in N-S direction &amp; torsion</td>
</tr>
<tr>
<td>5.18</td>
<td>2nd bending mode in E-W direction &amp; torsion</td>
<td>4.555</td>
<td>2nd bending mode in SW-NE direction</td>
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<tr>
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<td>torsion</td>
<td>4.759</td>
<td>2nd bending mode in SE-NW direction</td>
<td>5.379</td>
<td>2nd bending mode in SE-NW direction</td>
</tr>
</tbody>
</table>

Fig. 5 Calibration reconstructed bell tower

From the authors view [11] the primary objective, that the reconstruction measure significantly increases the bearing strength and that the eigenfrequency of the bells have a significant distance to the eigenfrequency of the tower could be proven.
Room for improvement is seen at the definition of the design space, the definition of the objective function as well as the optimization algorithm. Probably however also with increased complexity in the description of the objective function no significantly better calibration can be found in the design space. Facing the unknown mechanic properties of the masonry structures of historical bell towers it seems promising also to improve the FEM-modelling of the bell tower for properties whose calibration seems unsatisfactory. In order to verify the eligibility of the subspace for the identification sensitivity studies which consider all potentially differing mechanical properties are recommended.

4.2 Identification of material properties of visco-elastic adhesives in automotive engineering

For the simulation of visco-elastic adhesives for adhesive joints of electronic components material models are needed which can consider the dependency of mechanical properties of the adhesive joints from loading velocity and loading duration from a temperature range of –40 to 150 °C.

![Fig. 6 Adhesive joints of electric components](image)

For the adhesives DMA (dynamic mechanic analysis) master measurements as well as relaxation and creep tests were done. The measurements are done in the frequency range of 0.1 to 100 Hz as well as the temperature range from –40 to 150 °C. The material model for the simulation is described by a PRONY-series approach with 52 variables to identify overall. For computation of the experiments a mathematical model in a Fortran implementation with a small computation time was used. As objective function the integral sum of the quadratic calibration between measurement and computation were used. In addition energy conserving constraints have to be followed strictly. From the view of the authors [2] the optimization problem is characterized through a bad convergence of the calibration because of the large amount of local minima. For the calibration genetic optimization strategies were chosen. As is known genetic search strategies are suited to find as global optimums as possible in design spaces with lots of local optima. The identification ought to be used as automated routine for identification of the material characteristic by default. Most important requirement of the optimization algorithm therefore was to find as robust parameters as possible with an optimized adjustment of the evolutionary parameters for a good calibration for different adhesives. With a genetic optimization algorithm the boundary conditions could be fulfilled and a calibration was found which was better than the one found by hand.
Fig. 7 Calibration of measurement and computation using an automated genetic optimization strategy

By coupling of the genetic algorithm with a subsequent gradient based mathematical algorithm the calibration could be improved even further. Thereby the advantages of the genetic algorithm to get close to the global optimum in a robust way could be successfully combined with the advantage of the mathematic optimization algorithm to converge fast when started close to the optimum.

Fig. 8 Calibration between measurement and computation via automated genetic optimization strategy and subsequent gradient based mathematical

The automated identification of the PRONY-components could successfully be integrated into the normal process. Using the automated identification computation the variables for a material routine for the FEM program Abaqus for computation is given. Thereby the beforehand necessary operating expense of a stepwise calibration of single PRONY-coefficients by hand could be significantly reduced and a better calibration could be found.
4.3 Automated Validating of Airbag Models [3]

In order to secure the prognosis ability of numerical computation models of airbags the airbags are validated by computing several configurations and component experiments. In fig. 9 a configuration of an impactor test case and the resulting acceleration characteristic can be seen. The impactor test were done repeatedly with identical configuration and the scattering result values were averaged. The validation results from the expectation (mean value) of the experiment. The computation of the component experiments was done using MADYMO.

Abb. 9: Impactor test (left) und acceleration characteristics (right)

The design space of the calibration is described using six parameters, gas temperature, permeability as well as size and efficiency of airbag opening. Important evaluation parameters are the acceleration, the path and the pressure signal. Maxima and the points in time of the maxima as well as the temporal progression were observed. Beforehand a sensitivity study was carried out in which the matrix of linear coefficients of correlation was evaluated. The significant linear correlation between gas temperature, bag permeability and efficiency of the airbag opening to the other result variables is physical plausible and corresponds to the expectations. The other parameters of the design space show no significant connection to the result variables and therefore are not varied in the following identification. Furthermore it could be shown, that the averaged test curve lie within the set of curves of the sensitivity study. Therefore it can be assumed that a calibration with an as automatized optimization method as possible seems contingent.

Fig. 10 Grey - 100 computations of the sensitivity study, green – mean value curve of the experiment

The objective function of the optimization is assembled from the deviance of the acceleration run over time, the maximum value of the acceleration as well as with less weighting from the deviance of the pressure run over time [3]. Because of the insensitivity adverse local optima a genetic optimization algorithm was used. A population size of 10 individuals and 15 generations of 7 configurations.
resulted in a total of 1050 computations. With a simulation time of 3 minutes per configuration and 5 computations running parallel the whole optimization took 11 hours.

Figure 8 show the computation results of the optimal design compared to the experimental data. As well as the matching of the accelerations and the matching of the pressures is of high grade. The automated validation of the airbag models ought to be transferred to further airbag models and complete restraint systems. With increasing number of possible parameters of the design space the importance of the sensitivity study in order to reduce the parameters to identify and for validating the applicability of the design space for the sought after calibration increases. With increasing computation times furthermore ought to be checked how far adaptive response surface methods can contribute to reduction of the computation time.

5 Summary

Problems of identification and respectively calibration between measurement and simulation are processed in increasing manor using optimization algorithms. Hereby genetic and evolutionary strategies distinguish themselves through their robustness for global search of the best possible calibration. Gradient based optimization strategies are very useful, when they are started close to the optimum and therefore for the fine tuning of pre-optimized parameter sets. Response surface strategies further more offer an attractive alternative for small parameter spaces.

Besides the choice of the optimization algorithm the definition of the design space for the calibration search is of crucial importance for the identification problem. For this sensitivity studies are recommended, in order to identify as sensitive variables as possible and validate if they are accepted in the design space for which the calibration is made. Details on the definition of adequate objective functions and should the situation arise provides good starting designs for optimization.

For practical applications as robust settings of the optimization strategies and objective functions as possible are ought after. The calibration should be automated for repetitive problems and be repeatable with default-settings.

6 Literature


