Optimization of Hydrocarbon Production from Unconventional Shale Reservoirs using FEM based fracture simulation

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1 Introduction

Substantial quantities of oil and gas are currently being produced from Unconventional Resources / Reservoirs. These reservoirs are usually characterized by high shale content and ultra-low matrix permeabilities. Most completions in Unconventional Reservoirs are hydraulically fracture stimulated in order to establish a more effective flow from the far-field reservoir and fracture network to the wellbore. The success of hydraulic fracture stimulation in horizontal wells has resulted in it being ranked as one of the major distinguishing technologies of the 21st Century. It has already realized its potential to dramatically change the oil and gas production landscape across the globe, and the impact will endure for decades to come.

For a given field development project, the derived economics is highly dependent on the effectiveness of the drilling and completion operation to establish effective and retained contact with the hydrocarbon resource. This paper introduces a suggested process to model, calibrate, and optimize the landing of the well and the optimization of the hydraulic fracture stimulation design for naturally fractured reservoirs.

The introduced workflow combines the commercial software packages ANSYS® [1] and multiPlas [2] within a 3D Hydraulic Fracturing Simulator [3] for the parametric Finite Element (FEM) Modeling and material modeling of naturally fractured sedimentary rocks. With the utility of optiSLang® [4], automated sensitivity studies of the uncertainty of reservoir, engineering, and operational parameters are performed and are evaluated relative to the resulting Stimulated Reservoir Volume (SRV) and Accessible Hydrocarbon Resource. Results from these studies are then used to optimize the well placements and completion designs.

Unlike most academic and commercial approaches, the introduced approach uses a homogenized continuum approach to model the 3D hydraulic fracturing in naturally fractured reservoirs. The principal motivation for using a continuum approach is the numerical efficiency necessary to run fully 3D coupled hydraulic/mechanical simulations of the hydraulic fracturing of multiple stages and multiple wells in naturally fractured sedimentary rocks. A fully 3D discrete fracture simulator respective of Mohr-Coulomb failure is numerically quite intensive. A discrete fracture model has not yet been developed using a fully 3D explicit fracture growth modeling system.

Hydraulic fracturing in shale reservoirs is mostly dominated by the anisotropic stress and strength conditions resulting from the initial patterns of planes of weakness, these usually being the natural joints and fractures of the source rock. To capture this impact on fracture mechanics, the three-dimensional modeling of anisotropic strength, stress, and conductivity of the matrix and of the fracture system is required. Simulation simplification to 2D or pseudo-3D geometric modeling will fail to capture the effects necessary to properly model the potentially most important effects which may drive the hydraulic fracturing process and the resulting production performance.

The homogenized continuum approach was initially developed and applied in the Civil Engineering field of Waterway and Dam Engineering to better determine the influence of water flow in naturally fractured dam foundations [5]. It was improved and generalized for the coupled hydraulic-mechanical simulation of naturally fractured rocks using commercial FEM codes [6]. These developments provide
the basis for the software tool “multiPlas” [2], which provides for the non-linear modeling of jointed rocks.

The introduced workflow is an integrated well placement and completion design optimization workflow. The toolkit integrates geo-mechanical descriptions, formation characterizations, flow dynamics, microseismic event catalogues, hydraulic fracturing monitoring data, well completion and operational parameters in a modeling environment with optimization capability. It is built upon a 3D geological model with multi-disciplinary inputs including formation properties, in-situ stresses, natural fracture descriptions, and well and completion parameters (i.e., well orientation, landing interval, fluid rate and volume, perforation spacing, and stage spacing). Upon calibrating with the hydraulic fracturing field observations, the introduced workflow optimized well completion design, and guidance on data acquisition and diagnostic needs to achieve EUR performance at optimized costs.

1.1 Background on Dynardo’s Hydraulic Fracturing Modeling Approach

The inherent anisotropies of unconventional reservoirs result from layering, deformation history, strength and stress variability, and the non-uniform conductivity of the fractured rock mass. Because of these complexities, hydraulic fracturing should be simulated in a fully three-dimensional coupled hydro-mechanical model. Most shale hydrocarbon resources are essentially jointed even before hydraulic fracturing takes place. These planes of weakness include the bedding plane and usually two or three additional sets of natural planes of weakness.

Most commercial hydraulic fracture simulators model hydraulic fracturing using 1D, 2D, or pseudo 3D geometric approaches. In many cases, these simplifications prevent the simulators from adequately modeling the complex hydraulic fracturing mechanisms that are present. This may dramatically oversimplify the simulated fracture geometry, and may fail to identify the opportunities for economic production improvement in all but the most trivial of shale resource settings [7].

For a fully 3D modeling approach, achieving an effective numerical discretization capable of representing multiple stages and multiple wells in a complex reservoir setting is necessary. A discrete modeling approach of natural fractures or a homogenized modeling approach of fractures can be developed. However, the discrete modeling of a network of joints resulting from the hydraulic fracturing of the rock mass is currently computationally “extraordinarily expensive” to the point of impracticality for wellbore-scale models. Such models include discrete element and particle approaches (DEM, Particle codes) or discrete fracture modeling in continuum mechanics approaches (XFEM, cohesive zone elements). Currently, there are no commercial simulation solutions available for the wellbore-scale fully 3D hydraulic fracturing simulation of multiple stages and multiple wells using discrete joint modeling. Although a majority of research groups are following discrete fracture modeling approaches, a fully 3D discrete solution appears elusive at the needed wellbore-scale.

The modeling of coupled hydro-mechanical problems in rock mechanics using a homogenized continuum strategy was successfully implemented for science and industrial applications by Wittke [5] and others in the 1980s and 1990s. The weak point of the numerical implementation at that time was the inability of consistent integration of multi-surface plasticity, which is a result of dealing with multiple yield criteria at the material point level in the homogenized continuum representing intact rock (matrix) and multiple joint sets. To overcome the problem, Wittke introduced a pseudo-viscous numerical procedure which depended on pseudo-parameters which had a seemingly unreasonable influence on the results. Using a homogenized modeling approach for jointed rock in implicit integration algorithms resulted in convergence difficulties. Science groups moved to trial explicit time
integration procedures using discrete modeling techniques or particle based models. Here convergence problems are minimal. However, because of the stability requirements of explicit time integration schemes, these approaches became computationally “extraordinarily” time consuming when modeling transient 3D wellbore scale problems.

After attempting discrete joint modeling and explicit time integration methods for several years, researchers at the Bauhaus University in the late 90’s returned to a homogenized continuum approach, and developed a solution for the problem of consistent integration of multi-surface plasticity using implicit time integration [6]. As a result, hydraulic fracturing can now be efficiently modeled by using implicit finite element formulations, incorporating real world fully 3D reservoir conditions including all relevant anisotropies and thermo-hydro-mechanical coupling [8].

1.2 Every Hydraulic Fracturing Simulator Needs to be Calibrated for Typical Reservoir Conditions

A practical 3D Hydraulic Fracturing Simulator that could simulate multiple stages in multiple wells with reasonable numerical effort was now available. The challenge was then to properly characterize the geomechanical (stress, strengths, moduli, cohesion, friction angle, YM, PR, etc.) and the hydraulic (pressure, saturations, compressibility, permeability, etc.) setting of the resource. It is a formidable task to accurately measure the total state of the reservoir and bounding layers. However, in order to model realistic fracture height growths, all relevant potential fracture barriers need to be modeled and parameterized. Significant fracture barriers may occur due to layered contrast in deformation, stress, and strength characteristics. Similarly, faults and previously created hydraulic fractures may act as preferred mechanisms for fracture growth. These are generally to be expected when horizontal well fracture stimulations are closely staged.

After constructing a layered reservoir and bounding rock model inclusive of the potential fracture barriers, the calibration of large amounts of uncertain rock parameters to the best available measurements was necessary. A parameter identification problem exists simply because of the large number (>100) of model parameters, and they may have a considerable associated uncertainty. During the calibration phase, the workflow applies optiSLang [4], a commercial tool box for variation space management and optimization analysis. The process involves running a set of calibration models respective of the variation space of the model. With optiSLang, all parameters in a parametric hydraulic fracturing model can be identified and updated efficiently for successive model runs, which are then initialized and executed in an automated process. A large number of calibration sensitivity design runs can be executed in a comparatively short period of time.

The calibration phase ideally requires quality data measurements. This includes the pressure measurements that are used to derive ISIP/DFIT (Instantaneous Shut-In Pressure, Dynamic Fracture Intensity Test) conditions as well as the projected bottom-hole pressure history. The representative microseismic event catalog is also used in the calibration phase. Uncertainty analysis is integrated in the calibration process to better identify the most influential parameters controlling fracture geometry. This calibration process also provides the potential to focus additional data gathering to those parameters that significantly affect the simulation results.

Once a calibrated model is developed that is respecting of the resource data as well as the microseismic event data, the simulator can then be used in a forecast mode to better optimize the well landing depth and the completion design.
1.3 What is the Right Value to be Optimized?

After having a 3D Hydraulic Fracturing Simulator that is well calibrated to the reservoir, the next question is “What are the right quantities to optimize?” Conventionally, Stimulated Rock Volume (SRV) is used to quantify the effectiveness of the fracture stimulation. SRV variation is clearly dominated by fluid volume variation: more frac fluid simply creates more fractures and more frac volume. The simulation results can be used to fully quantify which reservoir layers are being fractured, how much frac height and frac length is produced, which fractures accept proppant based on apertures, and how the created fracture network might effectively drain the reservoir.

First, only created fractures that are acceptant of proppant with sustained connectivity to at least one perforation cluster or flow port are credited with production potential. The related proppant-accepting volume is referred to as **Valuable SRV (VSRV)**. Second, the **drainage volume** over the productive life of the well is calculated based on the VSRV with consideration for an average drainage radius. By integration of the pore and hydrocarbon content in the reservoir layers over the drainage volume, the **Accessible Hydrocarbon Initially In Place (AHCIIPI)** can be calculated. This represents the producible hydrocarbons connected to the wellbore. By applying a representative recovery factor, the estimated hydrocarbon production expected over the lifetime of the well is calculated. The economic maximization of AHCIIPI is usually the optimization goal.

A very important verification of the forecast quality of the calibrated reservoir model is the comparison of the estimated hydrocarbon production from the hydraulic fracturing simulator to Estimated Ultimate Recovery (EUR) of the calibration well and neighboring wells.

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**Fig. 1-1**  workflow for optimization of hydrocarbon production in unconventional oil and gas reservoir
During the optimization procedure, the major completion parameters such as well orientation, landing depth, stage design, well spacing, and fluid volume will generally be the most influential parameters. The optimization process is usually a compromise between increasing EUR potential subject to reducing completion costs. This optimization is represented by a classic Pareto Frontier. Using optiSLang, the Pareto Frontier represents the design limits where any production improvement cannot be introduced anymore without increasing the completion costs. The Pareto Frontier is the final result of the workflow. It is used for rationalizing the decision between maximizing AHCIIIP and minimizing the related completion costs.

2 The Dynardo Hydraulic Fracturing Simulator

The Hydraulic Fracturing Simulator [3] combines three commercial software packages: ANSYS® [1], multiPlas [2] and optiSLang® [4]. ANSYS is used for the development of parametric reservoir finite element models. The coupled hydraulic-mechanical analysis is performed with ANSYS. The second module, multiPlas, is an ANSYS extension for non-linear material modeling. These material models extend the ANSYS functionality to the non-linear mechanical analysis of naturally fractured rocks. Within the context of the hydraulic fracturing simulator, multiPlas additionally provides an anisotropic hydraulic element which models the flow through fractured rock. The third software product, optiSLang, is used to efficiently calibrate the model and to perform sensitivity analyses in consideration of the uncertainties in the reservoir model and operational conditions.

In order to reach a sufficient forecast quality in the simulation of hydraulic fracturing, the most important phenomena that needs to be represented by the model is the three-dimensional anisotropic strength and conductivity distribution in naturally fractured sedimentary rocks. In the case of unconventional gas and oil shale, the rock is classified as jointed rock having isotropic “intact” rock strength and multiple sets of planes of weakness. For the purpose of this document, these planes of weakness are called “joint sets”. Obviously the sedimentary rock has a bedding plane (see Fig. 2-1, label “Sch”). There are often two additional sets of strength anisotropies (see Fig. 2-1, labels “K1, K2”). In some reservoir layers these joint sets are either open or cemented/healed.

![Fig. 2-1 Jointed rock characterization and translation to homogenized continuum approach](image)

In some instances, these joint sets can be identified on various open hole logs or cores. In other instances, these joint sets are closed and the identification of the joint sets in open hole logs or cores might be difficult.

The fracture simulator applies the continuum approach which is based on the concept of homogenization. In contrast to discrete models, the joint sets are not explicitly modeled as geometry boundaries. The influence of the joint sets is explicitly taken into account within the anisotropic strength model of jointed rock which results in an anisotropic conductivity development in the event
of the rock failure represented by the element. Essentially, a joint set dilates opens, and the associated conductivity increases due to either an oriented tensile or an oriented shear failure. At the conclusion of the frac job, the net pressure decline may result in the joint set aperture reducing, resulting in a reduction of the associated conductivity. Both of these effects are taken into account in the simulator.

In the simulation, the tensile and shear failure modes of intact rock and of the individual joint sets are consistently treated within the framework of multi-surface plasticity [9]. The multi-surface strength criterion is evaluated at every discretization point in space. If the stress state violates the multi-surface yield criterion, then plastic strains develop and strength degradation occurs. By introducing “mean effective” activated joint set frequencies that can be defined for every joint set and for every individual layer, the homogenized joint openings and the corresponding joint conductivities can be calculated based on the plastic strains. The individual values can be evaluated and visualized in the post-processing step. The initial natural frequency of the planes of weakness and the mean effective activated frequency of stimulated joints will usually vary. As a result, determining the activated average frequency of joints is an important undertaking in the calibration process.

The homogenization approach can and should be coupled with discrete anisotropies such as major faults if the dimension of the discrete anisotropies are large compared to the overall modelled 3D geometry or if discrete effects at major faults are of interest. The fault is modelled as discrete 3D geometry feature, and an oriented joint set is used to define the shear and tensile strength criteria of the fault.

### 2.1 Parametric Reservoir Modeling

The simulation of hydraulic fracturing requires the calibration of important but somewhat uncertain parameters. The reservoir system, inclusive of the wellbores and the frac stages, should be parametrically modeled in order to allow for an efficient calibration procedure. The entire process of model generation (pre-processing), model solution, and model post-processing should ideally be an automated process. The hydraulic fracturing simulator offers a predefined parametric representation of the following inputs:

1/ **Model geometry**: number of wells/stages, stage positions and orientations, number of perforation clusters per stage, distance between perforations, distance between stages, well/stage depths, horizontal well orientation, number and depth of all rock units

2/ **Finite element mesh**: definition of model boundaries, definition of volumes with different element size (e.g. fine mesh at perforations and coarse mesh at the model boundary), element size, type of mesh, type of coupling, perforation size

3/ **Initial stress field**: piece-wise linear distribution (linear inside one layer, but jumps at the boundary between two layers) of total vertical stress, minimum horizontal effective stress (k0-values) and maximum horizontal effective stress, direction of minimum horizontal stress

4/ **Initial pore pressure field**: piece-wise linear distribution of pore pressure (linear inside one layer, but jumps at the boundary between two layers)

5/ **Material properties of all rock layers**: linear and nonlinear mechanical material properties including definition of up to four joint sets, hydraulic material properties

6/ **Well treating**: slurry rate and bottom hole pressure as a function over time, average proppant size and proppant pumping, fluid viscosity, perforation conductivities

7/ **Coupling parameters**: average activated joint set distance, joint set roughness coefficient, stress dependency of joint conductivity
Simulation parameters: time stepping, post-processing

The parametric modeling approach is derived from the ANSYS internal programming language APDL.

Most of the parameters are separately defined for each distinctive rock layer and for each joint set. Several hundred parameters are generally required for a model run. As part of the parameter definition/selection processing: the automatic generation of the finite element model, the calculation of in-situ reservoir conditions, and the well design and the operational conditions are all tested for consistency before a unique model execution begins. On occasion, a parameter selection is made for a specific model run that results in an unrealistic (unstable) initial condition. When this occurs, these unstable models are identified and their run time execution is terminated.

2.2 Coupled Hydraulic-Mechanical Analysis

Hydraulic fracturing is a coupled hydraulic-mechanical problem. In the hydraulic module, the pressure increases in the fracture initialization location due to the pumping of fluid and low initial rock permeability. Within the homogenized continuum approach, pressure is treated as “pore pressure” representing the pressure in the fracture network. In the mechanical part the increase of pressure modifies the effective stresses acting on the rock. If the pressure is large enough, the jointed rock fails and fractures start to open. As a result, the rock permeability increases, which directly influences the pressure distribution in the hydraulic module.

Fig. 2-2  Schematics of 3D coupled hydraulic-mechanical simulation

In the simulation of hydraulic fracturing, these primary coupling effects need to be resolved. Fig. 2-2 shows a flowchart of a coupled hydraulic-mechanical analysis with the Dynardo hydraulic fracturing simulator.

The automatic simulation procedure starts with the setup of the reservoir geometry based on a set of input parameters representing the layering, the well, and the stage design. Then the finite element models are generated and the in-situ conditions are applied. In the hydraulic model, the pore-pressure field is initialized with the initial reservoir pore pressure conditions. The mechanical model is initialized with the initial effective stress distribution. A non-linear mechanical analysis is performed.
to ensure consistency between the mechanical parameters and the initial stress field. The initial conditions should not result in plastic strains in the model.

After model initialization, the actual simulation cycle for hydraulic fracturing starts. In each cycle, the hydraulic and the mechanical sub-model are independently solved. The coupling between the models is realized by an update of material parameters and loading conditions in the corresponding sub-models. The following couplings are applied:

1/ **Stress state update (hydraulic-mechanical coupling):** based on the pore-pressure distribution in the hydraulic model, flow forces are applied in the mechanical analysis.

2/ **Fluid material properties update (mechanical-hydraulic coupling):** based on the plastic strain and the stress distribution in the mechanical model, the conductivities are updated in the hydraulic model. Because of the anisotropic failure of the joint sets, an anisotropic conductivity tensor is obtained.

The coupling is performed in an explicit way. Consequently, one iteration cycle is performed for every time step. The time step needs to adequately represent the progress of the fracture growth. The cycle starts with the transient hydraulic analysis. The pore-pressure field is updated and the corresponding flow forces are calculated and applied to the mechanical model. The next step is the nonlinear mechanical analysis which results in a new stress and plastic strain distribution. The resultant update of the hydraulic conductivities is applied to the hydraulic model in the subsequent time-step.

### 2.3 Non-Linear Mechanical Analysis

In the mechanical sub-model, a nonlinear static finite element analysis, cf. [10], is performed. The nonlinearities are caused by failure of the material. In ANSYS, the nonlinear constitutive behavior of jointed rock is described with the external library multiPlas [2]. By using the ANSYS “usermat” API for user-defined material models, multiPlas provides nonlinear material models for typical materials in geomechanical and civil engineering studies.

The mechanical analysis of jointed rock incorporates the concept of effective stresses. This is the stress which directly acts on the rock and which results in a deformation of the rock. The effective stress tensor $\sigma_{\text{eff}}$ are defined as:

$$
\sigma_{\text{eff}} = \sigma_{\text{tot}} - pI
$$

(2.1)

where $\sigma_{\text{tot}}$ is the total stress tensor, $p$ is the pore-pressure and $I$ is the second order identity tensor.

The homogenized continuum approach is applied to describe the deformation behavior of jointed rock. Consequently, the stress-strain relationship does not describe the deformation behavior of the individual constituents, intact rock and joint sets, but the overall response of the homogenized jointed rock mass. The corresponding linear-elastic stress strain relationship can be written as:

$$
\sigma_{\text{eff}} = D : \varepsilon
$$

(2.2)

where $D$ is the generally orthotropic linear elastic material tensor of the homogenized rock mass and $\varepsilon$ is the strain tensor.

In multiPlas, the description of the nonlinear behavior of jointed rock is based on the concept of rate-independent plasticity, cf. [9] [11]. It is assumed that the total strain $\varepsilon^{\text{tot}}$ can be decomposed into an elastic part $\varepsilon^{el}$ and a plastic part $\varepsilon^{pl}$:
\[ \varepsilon^{\text{tot}} = \varepsilon^{\text{el}} + \varepsilon^{\text{pl}}. \]  

The stresses are related to the elastic strains by the linear elastic material matrix. Consequently, Eq. (2-2) can be rewritten as:

\[ \sigma_{\text{eff}} = D : \varepsilon^{\text{el}}. \]  

The plastic strains develop if a certain strength criterion, conventionally referred to as the yield condition, is violated. In this context, the boundary of the admissible stress space (elastic domain) is called yield surface.

The strength of the homogenized jointed rock is defined by the strength of the individual constituents. As a result, the overall strength criterion is not a smooth surface, but is composed of multiple yield surfaces. Each yield surface represents a specific failure mode of one of the constituents.

In the multiPlas material model for jointed rock, isotropic strength is assumed for intact rock. Two fundamental failure modes are considered. Tensile failure of intact rock is represented by the Rankine yield surface. The corresponding yield condition can be written as:

\[ F_{\text{RK},I} = \sigma_{1} - f_{t,I} \leq 0, \]  

where \( \sigma_{1} \) is the maximum effective principal stress (tensile stresses are positive) and \( f_{t,I} \) is the uniaxial tensile strength. Shear failure of intact rock is described by the Mohr-Coulomb yield condition, which reads:

\[ F_{\text{MC},I} = \frac{\sigma_{1} - \sigma_{3}}{2} + \frac{\sigma_{1} + \sigma_{3}}{2} \sin \varphi_{I} - c_{I} \cos \varphi_{I} \leq 0, \]  

where \( \varphi_{I} \) is the intact rock friction angle, \( c_{I} \) the cohesion, \( \sigma_{1} \) is the maximum effective principal stress, and \( \sigma_{3} \) is the minimum effective principal stress.

The multiPlas material model currently allows the definition of up to four joint sets. In contrast to intact rock, the joint strength criteria are anisotropic. The strength criteria depend on the joint orientation, which is described by the strike angle \( \alpha \) and the dip direction \( \beta \). The corresponding yield surfaces are defined in terms of the normal joint stress \( \sigma_{N,J} \) and the in-plane shear stress \( \tau_{J} \). Both stress components are obtained by rotating the global stress tensor into the local joint coordinate system. Similar to intact rock, two failure modes are taken into account for every joint set. The tension cut-off yield surface represents tensile failure normal to the joint. The corresponding yield condition reads:

\[ F_{T,J} = \sigma_{N} - f_{t,J} \leq 0, \]  

where \( f_{t,J} \) is the tensile strength of the joint set. Joint shear failure is described by the Mohr-Coulomb yield surface:

\[ F_{\text{MC},J} = \tau_{J} + \sigma_{N} \tan \varphi_{J} - c_{J} \leq 0, \]  

where \( \varphi_{J} \) is the joint friction angle and \( c_{J} \) is the joint cohesion.

The individual yield surfaces of the multiPlas jointed rock material model are visualized in Fig. 2-3. If in the simulation a strength criterion becomes active, the corresponding strength parameters are reduced to residual values. Dilatancy effects are taken into account for shear failure by incorporating non-associated flow rules. The corresponding plastic potentials are obtained from the Mohr-Coulomb conditions by replacing the friction angle with the dilatancy angle, cf. Eqs. (2-6) and (2-8).
2.3.1 Consistent Numerical Treatment of Multiple Strength Conditions

The non-linear behavior of jointed rock is described by a set of different strength conditions. As a result, the boundary of the admissible stress space becomes non-smooth which requires a special numerical treatment. In multiPlas, the multi-surface plasticity approach, introduced by [9], is implemented which allows for an efficient and consistent treatment of multiple yield conditions.

In the multi-surface plasticity approach, the plastic strain increment is defined by a modified flow rule which can be written as:

$$\Delta \varepsilon^{pl} = \sum_{\alpha=1}^{n_{yc}} \Delta \lambda^\alpha \mathbf{g}^\alpha$$  \hspace{1cm} (2.9)

where $n_{yc}$ is the number of yield conditions, $\Delta \lambda^\alpha$ is the plastic multiplier and $\mathbf{g}^\alpha$ is the direction of plastic flow of yield condition $\alpha$. A stress state is admissible if all yield conditions are satisfied. If the stress state is on a yield surface, then plastic strains develop for that yield surface. Because the flow rule defines an oriented direction of plastic flow, the corresponding plastic multiplier must be positive. Any stress state must satisfy these conditions, which are known as Kuhn-Tucker or loading/unloading conditions, for each yield criterion:

![Jointed rock yield surfaces of intact rock and joint sets in multiPlas](image)
\[ F_a \leq 0 \quad F_a \Delta \lambda^a = 0 \quad \Delta \lambda^a \geq 0 \quad \alpha = 1 \ldots n_{YC}. \quad (2-10) \]

Consequently, in a plastic step, the stress state might be located on more than one yield surface. This is illustrated in Fig. 2-4 for a two surface model. In order to handle the singularity at the intersection between both yield surfaces, the stress state must satisfy both conditions. As a result, the direction of plastic strain is defined as a combination of the individual directions.

In the numerical implementation, the stress-calculation is performed in two steps. In the first step, a trial stress state is calculated assuming that the plastic strain obtained in the previous step does not change. The yield conditions are evaluated for this trial stress state. A set of active yield surfaces is defined by all yield conditions which are violated by the trial stress state. If the set of active yield surfaces is empty, the trial stress state is admissible. Otherwise, the trial stress needs to be returned to all active yield surfaces. In this second step, the standard return mapping algorithms, i.e., cutting plane or closest point projection, are applied. In contrast to the classical single-surface plasticity, the return mapping algorithm must simultaneously handle multiple yield surfaces which results in a system of generally nonlinear equations. An additional activity condition is introduced. A yield condition is removed from the set of active yield surfaces if the corresponding plastic multiplier becomes negative during the iteration.

2.4 Hydraulic Analysis

In the hydraulic step, a transient analysis is performed. In order to cover gravity effects, the governing equations are not expressed in terms of the pore-pressure, but rather in terms of the hydraulic head. The hydraulic head \( h \) of a fluid is defined as the combination of the pressure head and the elevation head:

\[ h = \frac{p}{\rho g} + z, \quad (2-11) \]

where \( p \) is the pore-pressure, \( \rho \) is the fluid density, \( g \) is the standard gravity and \( z \) is the elevation.

The analysis is based on the groundwater flow equation:

\[ S_s \frac{\partial h}{\partial t} = -\nabla q + R \quad (2-12) \]

where \( S_s \) is the specific storage, \( R \) is a general source and sink term, and \( q \) is the flux vector. The specific storativity is one of the most important hydraulic parameters that needs to be calibrated for the
reservoir. The storativity represents the amount of stored energy in open joints, and is related to the energy losses due to friction or of leakage during the hydraulic fracturing process.

Similar to the mechanical model, the continuum theory is applied in the hydraulic model. As a result the flux vector can be related to the hydraulic head by Darcy’s law:

\[ q = -K \nabla h \]  

(2.13)

where \( K \) is the conductivity matrix of the jointed rock.

\[ \begin{align*}
K &= K_I + \sum_{j=1}^{n_J} K_J^{(j)} \\
K_I &= \frac{\rho g}{\mu} \begin{bmatrix} k_{\text{ini},h} & 0 & 0 \\
0 & k_{\text{ini},h} & 0 \\
0 & 0 & k_{\text{ini},v} \end{bmatrix}
\end{align*} \]

(2.14)

Fig. 2-5  Darcy flow equation in homogenized continuum mechanics

As shown in Fig. 2-5, the Darcy equation describes the flow through the homogenized jointed rock. The hydraulic conductivity matrix \( K \) represents the overall conductivity of the rock including all joint sets. The homogenized conductivity is obtained by superimposing the contributions of the individual constituents:

\[ K = K_I + \sum_{j=1}^{n_J} K_J^{(j)} \]

(2.14)

where \( K_I \) is the hydraulic conductivity of intact rock, \( n_J \) is the number of joint sets, and \( K_J \) is the hydraulic joint set conductivity. In the simulator, the intact rock conductivity represents the initial rock conductivity. By assuming a transversely isotropic behavior, the intact rock conductivity matrix is given by:

\[ K_I = \frac{\rho g}{\mu} \begin{bmatrix} k_{\text{ini},h} & 0 & 0 \\
0 & k_{\text{ini},h} & 0 \\
0 & 0 & k_{\text{ini},v} \end{bmatrix} \]

(2.15)

where \( \rho \) is the fluid density, \( g \) is the standard gravity, \( \mu \) is the dynamic fluid viscosity, \( k_{\text{ini},h} \) is the initial horizontal rock permeability, and \( k_{\text{ini},v} \) is the initial vertical rock permeability. Failure of intact rock does not change the initial rock conductivity matrix. As shown in section 2.5.2, intact rock failure is handled by introducing additional joint sets. In the local joint coordinate system, the joint set conductivity matrix is given by:

\[ K_j = \frac{\rho g}{\mu} k_j \begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \end{bmatrix} \]

(2.16)
where $k_j$ is the in-plane joint permeability. In the initial state the joint permeability is zero. If a joint set fails, the joint opens up and the joint permeability increases. This relationship is described in detail in section 2.5. The global joint conductivity matrix is obtained by rotation of the local matrix:

$$K_j = R^T K'_j R,$$

(2-17)

where $R$ is a matrix describing the rotation from the global into the local joint coordinate system. In the global coordinate system, the joint conductivity matrix is generally anisotropic. As a result, the homogenized conductivity matrix $K$ becomes anisotropic during the simulation.

By substituting Eq. (2-13) into Eq. (2-12) the transient seepage equation is obtained:

$$S_e \frac{\partial h}{\partial t} = -\nabla (K \nabla h) + R$$

(2-18)

This equation is solved by using finite element techniques. Equation (2-18) is analogous to the heat equation in heat transfer problems. ANSYS heat transfer elements seemingly could solve the problem. However, because of the anisotropic hydraulic conductivity matrix, Dynardo implemented a new hydraulic element that more effectively manages the anisotropy.

### 2.4.1 Well Treatment and Pipe Modeling

In the hydraulic model, the reservoir inclusive of the perforations are modelled by solid elements. Additional 1-D pipe elements are introduced to connect the perforations of one stage to the volume elements. Fig. 2-6 shows the pipe definition in the model. The red line represents the well bore which connects the perforations. The hydraulic properties of the well bore are defined by the pipe diameter and the pipe conductivity. In general a large conductivity value is applied for the well bore. The green lines are the equivalent perforation pipes/tunnels that connect the well bore with the center of the reservoir volume elements. The perforation pipes are introduced to model a pressure drop between the well and the end of perforation. The hydraulic conductivity of the perforation pipes are defined in terms of a prescribed pressure drop relation:

$$K_{perf} = \frac{4 \rho g L}{\pi d_{perf}^2 \Delta P} \frac{Q_{ref}}{n_{perf}}$$

(2-19)

where $L$ is the pipe length, $d_{perf}$ is the pipe diameter, $Q_{ref}$ is the reference slurry rate, and $n_{perf}$ is the number of perforations. The pipe elements are automatically created during the model generation process.

*Fig. 2-6  Slurry Rate boundary condition*
prescribed BHP on perforations

Fig. 2-7 Bottom Hole pressure boundary condition

In the simulator, the loading conditions are applied either to the well pipe or to the perforation pipe. Two types of loading conditions are supported.

An inflow condition is defined in terms of a prescribed slurry rate. By applying the slurry rate (SR) to the well pipe, as shown in Fig. 2-6, the slurry in the perforation (the outflow from perforation to the reservoir) is defined by the conductivity buildup in the rock connected to the perforation.

Alternatively, a pressure condition can be applied to define bottom-hole pressure (BHP) conditions. During the model calibration, pressure conditions are used to model and verify ISIP/DFIT conditions. In that context, the measured BHP pressure is applied directly to the perforation pipe. Fig. 2-7 shows that in that case the pressure is prescribed at the nodes at the intersections between perforation pipes and well pipe. Using predefined pressure conditions, the user should disable the connection between the perforations by reducing the well pipe conductivity to a small value.

2.5 Mechanical-Hydraulic Coupling

According to Reference [5], the joint set permeability (hydraulic model) is related to the joint opening (mechanical model). If a joint opens up, then the permeability increases. In the mechanical analysis, the development of fractures is represented by a plastic material model. As a result, the joint set opening is not directly measured but needs to be calculated based on the plastic strains. Additional history variables are introduced which monitor the normal plastic strains of every joint set during the mechanical analysis. Both failure modes, tensile and shear, result in a normal plastic strain component. The amount of normal plastic strain due to shear failure can be controlled by the dilatancy angle. For a specific joint set, the normal plastic strain increases only if the corresponding yield surfaces are active. The mechanical (geometrical) joint opening of a joint set $E$ is defined as:

$$E = \epsilon_{N}^{pl} S$$

(2-20)

where $\epsilon_{N}^{pl}$ is the normal plastic joint strain and $S$ is the average activated joint set distance. The activated joint set distance is an input parameter and needs to be calibrated. If the activated joint set distance becomes larger than the element size, in order that the continuum theory remains valid, the activated joint set distance is limited by an equivalent element length $l_{eq}$:

$$S \leq l_{eq}.$$ 

(2-21)

The equivalent element length is a one-dimensional measure for the size of the domain represented by an integration (material) point. According to Reference [12], the equivalent element length $l_{eq}$ for an 8-node brick element with 8 integration points can be defined as:

$$l_{eq} = \frac{3}{\sqrt{8}} V_e$$

(2-22)
where $V_e$ is the element volume.

In the original derivation of the joint set permeability in Reference [5], a laminar flow between two smooth planes is assumed. In reality, the joint surface is neither planar nor smooth. Consequently, the mechanical opening must be related to the effective hydraulic opening of the idealized joint set [13] [14]. In the simulator, the following relationship is applied:

$$e = \frac{E}{r_{EE}} \quad (2-23)$$

where $e$ is the effective hydraulic opening and $r_{EE}$ is a prescribed ratio between both opening measures. In most applications of the simulator, a ratio between 1 and 2 is used initially, and later adjusted and verified during the calibration process.

The relationship between the effective hydraulic opening and the hydraulic joint set permeability is given by a cubic law:

$$k_j = \frac{e^3}{12.5 R_C} \quad (2-24)$$

where $R_C$ is the joint roughness coefficient. This relationship is visualized in Fig. 2-8. In order to be able to limit the flow in the joint set, a maximum effective hydraulic opening, $e_{max}$, is introduced. This maximum hydraulic opening results in the maximum hydraulic conductivity, and is related to the in-situ stress, the fluid, and the proppant placement condition. A limitation to this value can usually be seen in experimental data. This parameter is one of the most important model parameters that should be properly calibrated.

Fig. 2-8  Coupling between joint set permeability and joint set opening

2.5.1 Stress Dependent Fracture Openings

Since the joint opening is described by a plasticity model, the closure of joints, i.e., the reduction of normal plastic joint strains, is not represented in the mechanical model. The effect of compressive normal joint stresses on the joint set permeability is not taken directly into account in Eq. (2-24). As shown in Reference [14], this effect can be observed in experiments and will have a significant influence on the resulting joint conductivity during production. The simulator optionally allows for
this effect to be managed. If the stress dependency is enabled, then the joint set permeability is calculated as:

\[ k_J(e, \sigma_N) = f(\sigma_N)k_{J0}(e), \quad (2-25) \]

where \( k_{J0} \) is the stress independent joint set permeability given by Eq. (2-24), \( f \) is a dimensionless scaling factor ranging from a minimum value to 1, and \( \sigma_N \) is the normal joint stress. Based on [15] the following stress dependency function is implemented:

\[
f(\sigma_N) = \begin{cases} 
1 & \sigma_N > 0 \\
(1 - f_{\text{min}}) \left[ 1 - \left( \frac{\sigma_N}{D} \right)^n \right]^2 + f_{\text{min}} & D \leq \sigma_N \leq 0, \\
f_{\text{min}} & \sigma_N < D 
\end{cases} \quad (2-26)
\]

where \( D \) is the limit compressive stress (negative), \( f_{\text{min}} \) is the minimum scaling factor, and \( n \) is a shape factor. Fig. 2-9 visualizes the influence of that shape factor. For the post-processing of the joint openings, the openings are recalculated by introducing the stress dependent joint set permeability into the cubic law, Eq. (2-24).

**Fig. 2-9 Joint hydraulic conductivity for stress dependent part**

The conductivity decline function (stress dependency function) is additionally influenced by the proppant placement in the fractures. In general, higher pressures are required to close a fracture which is filled with proppant than a fracture without proppant. This effect is taken into account by defining two different stress dependency functions, namely limit stress and minimum scaling factor. The stress dependency function for joints with proppant is applied in all elements having proppant-accepting mechanical joint openings and which are connected to perforation clusters with elements having all proppant-accepting joint openings, cf. Section 2.7.1. In all other elements, the stress dependency function for joints without proppant is used. Usually the stress dependency parameters are derived from lab tests of conductivity at varying proppant concentrations and normal stress conditions.
2.5.2 Influence of Intact Failure on Hydraulic Conductivity Tensor
In addition to joint failure, the intact rock might fail as well, and the hydraulic conductivity of the jointed rock increases. In order to capture this phenomenon, up to three additional joint sets, one for tensile failure and two for shear failure, are introduced in case of intact rock failure. These additional joint sets are introduced if the corresponding intact rock failure criterion is violated for the first time. In the case of tensile failure where the Rankine yield surface becomes active, the additional joint is oriented perpendicular to the maximum principal stress direction. In the case of shear failure where the Mohr-Coulomb yield surface becomes active, the orientation of two additional joint sets coincides with the orientation of the shear failure planes in that step. After initialization of the additional joint sets, the orientation is fixed for that element for the duration of the simulation. For these additional joint sets, the mechanical-hydraulic coupling is performed in the same way as for the pre-defined joint sets.

From experience in shale reservoirs, the hydraulic conductivity change primarily from intact failure occurs in fracture barriers, which usually represents reservoir layers without vertical joint sets.

2.6 Hydraulic-Mechanical Coupling
Fluid flow in joints results in normal forces and shear forces at the joint walls [5]. The flow forces are related to the pore-pressure gradient. In the global orientation, the flow force vector $f_{ff}$ acting on the element volume (body force) can be written as:

$$f_{ff} = \rho g I$$

where $\rho$ is the fluid density, $g$ is the standard gravity, and $I$ is the gradient of the hydraulic head. The corresponding nodal force vector is obtained by integration of the flow force vector over the element volume. The individual nodal contributions are assembled and transferred to the mechanical model. Because of the incremental solution procedure, only the variation in the flow forces is added to the nodal forces in the mechanical model at every time step.

2.7 Post Processing
In addition to the traditional ANSYS post-processing functionality, e.g., stress plots, the simulator provides additional hydraulic fracturing specific outputs. These additional post-processing features are provided as parameterized APDL macros. In order to reduce the amount of data which is produced during the simulation and in order to reduce the total simulation time, the frequency of post-processing steps is also parameterized. The additional post-processing includes:

- bottom hole pressure and slurry rate over time (per perforation and per stage)
- fluid and fracture volume balance, e.g. fluid inflow and created joint volumes over time
- plots of joint set openings, joint set conductivities
- pore-pressure plots
- plots of the stimulated rock including microseismic events (all plastic elements, connected water-accepting plastic elements and connected proppant-accepting plastic elements) and the corresponding stimulated rock volume over time
- plastic activity over time
- connected drainage volume
- fracture extension compared to microseismic events

In addition to this predefined post-processing macros, all results can be exported into ASCII files.
2.7.1 Calculation of Connected Water and Proppant-Accepting Volume
Based on the mechanical joint openings, elements are identified as water-accepting or as proppant-accepting. An element becomes water-accepting if the mechanical opening of at least one joint set exceeds a predefined threshold. This threshold is parameterized. Usually a threshold of 0.1 mm is applied. A proppant-accepting element is identified if the mechanical opening of at least one joint set exceeds a multiple of the average proppant size. The factor and the average proppant size are also parameters of the model. In most of the reservoirs, a threshold of 3 times the average proppant size is applied.

In addition to the water and proppant-accepting elements, the corresponding connected sets of water and proppant-accepting elements are identified. An element is part of the set of connected water-accepting elements if the fluid can flow from any perforation into that element only by flowing through the other elements in that set. The sets of connected water and of connected proppant-accepting elements are continuously updated during the simulation. At the beginning of the simulation, the perforation elements are added to the connected sets. After every mechanical step, the water and proppant-accepting elements are identified. Based on the connected sets from the previous step, the neighbouring water or proppant-accepting elements are selected and added to the corresponding connected set. Two elements are neighbours if they are connected by at least one node. This selection algorithm is continued until no new neighbour elements are found. The sets of connected elements are history dependent.

For connected proppant-accepting volumes, the possibility of successful proppant placement is presumed. If proppant is placed in the fractures, it has an influence on the conductivity decline function, cf. Section 2.5.1. The stress dependency function for joints with proppant is only used for elements which are part of the set of connected proppant-accepting volume. Otherwise the stress dependency function for joints without proppant is applied even if the opening is larger than the proppant-accepting opening threshold.

2.7.2 Calculation Connected Drainage Volume
Based on the set of connected proppant-accepting elements, the drainage volume can be calculated. The drainage volume is defined by all elements which can be drained during the production time of the well from the set of connected proppant-accepting elements. The corresponding elements are identified by selecting, from the set of “connected proppant-accepting elements,” all elements which satisfy the following criteria:

- The element is in the same element layer of the layered reservoir as the connected proppant-accepting element. This is based on the assumption that only the “horizontal” initial permeability of unstimulated rock provides a mechanism for flow through unstimulated rock, this horizontal permeability being several orders of magnitude larger than the effective vertical permeability.
- The distance between the element center and the center of the proppant-accepting element is less than the drainage radius.
3 Application to North American Reservoir

3.1 Milestones and Goals

After several years of field development, the standard completion practices in the Reservoir were investigated for the potential to improve hydrocarbon production. The workflow was applied. The hydraulic fracturing simulator was calibrated to a well with a suitable set of whole core and log data, as well as quality microseismic. After calibration, a sensitivity analysis to possible variations of operational conditions was performed, and Meta-Models of the Optimal Prognosis (MOP) was derived. These Meta-Models were derived from multiple simulation results. They represented a verified correlation between all of the inputs to the model and the simulation results. The forecast quality of the MOP’s was verified during the calibration phase and confirmed with offset well production performance. The Meta-Models were then used in a fully predictive mode to optimize the completion design subject to economic considerations (e.g., maximum VSRV, AHCIIIP, hydrocarbon production, costs).

3.2 Modeling of the Calibration Well and the Calibration Stages

One well out of a pad of four wells was chosen as the calibration well. It was the first well on the pad that was completed. When considering the effects of fracturing induced inter-stage stress shadowing, the modeling of three successive stages appeared adequate.

Model geometry definition used parametric assignments regarding positions of stages, number of stages, number of perforation clusters, distance between perforations and stages, definition of model boundary and fine-coarse mesh boundary, well depth, horizontal well orientation, depth of all rock units, etc.

![Stratigraphic column of all modelled layers of the reservoir model](image)

*Fig. 3-1  Stratigraphic column of all modelled layers of the reservoir model*
For the parametric mechanical and hydraulic grid meshing, only brick elements were used. The Hydraulic Mesh is 8 times finer (i.e. a single mechanical element volume is presented by 8 hydraulic elements at 2 x 2 x 2) to sufficiently capture the pore pressure gradients. Elements with high aspect ratios were avoided to reduce mesh influence to the fracture growth. Sizes of fine-mesh and coarse-mesh volumes were input parameters of the parametric model. Size of volume for fine mesh was selected to capture all microseismic events inside the fine mesh volume (Fig. 3-2).

**Fig. 3-2** Microseismic location of the three modelled stages - top view

**Fig. 3-3:** FE-Model with stage 1, 2, 3

**Fig. 3-4:** Mesh for mechanical analysis
### 3.3 Definition of Reservoir Parameters

Initial elastic properties of layers and UCS values were taken from log and core data analysis. Initial strength properties of intact rock were derived from UCS values. Intact rock tensile strength was initially assumed to be 10% of UCS for all layers. Defining the friction angle of intact rock initially to 45°, cohesion values were then calculated.

Based on micro-seismic data, core, and log data interpretation, the expected fracture barriers were located in two different layer. The barriers were initially modelled without initial vertical joint sets. Model boundary layers where no fracture penetration was seen in the microseismic surveys were modelled with elastic material properties. Initial joint set strength parameters were derived from experience with other reservoirs. The fractured rock properties are shown in Table 3-1.

#### Table 3-1: Strength definition of intact rock and joint sets

<table>
<thead>
<tr>
<th>Layer</th>
<th>Intact Rock</th>
<th>Bedding Plane</th>
<th>Vertical Joint Set - 1</th>
<th>Vertical Joint Set - 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>45.0</td>
<td>4142.1</td>
<td>2000.0</td>
<td>20000.0</td>
</tr>
<tr>
<td>L2</td>
<td>45.0</td>
<td>4349.2</td>
<td>2100.0</td>
<td>21000.0</td>
</tr>
<tr>
<td>L3</td>
<td>45.0</td>
<td>3624.4</td>
<td>1750.0</td>
<td>17500.0</td>
</tr>
<tr>
<td>L4</td>
<td>45.0</td>
<td>8802.0</td>
<td>4250.0</td>
<td>42500.0</td>
</tr>
<tr>
<td>L5</td>
<td>45.0</td>
<td>3603.7</td>
<td>1740.0</td>
<td>17400.0</td>
</tr>
<tr>
<td>L6</td>
<td>45.0</td>
<td>3189.4</td>
<td>1540.0</td>
<td>15400.0</td>
</tr>
<tr>
<td>L7</td>
<td>45.0</td>
<td>3175.8</td>
<td>1630.0</td>
<td>16300.0</td>
</tr>
<tr>
<td>L8</td>
<td>45.0</td>
<td>3189.4</td>
<td>1540.0</td>
<td>15400.0</td>
</tr>
<tr>
<td>L9</td>
<td>45.0</td>
<td>3189.4</td>
<td>1540.0</td>
<td>15400.0</td>
</tr>
<tr>
<td>L10</td>
<td>45.0</td>
<td>2050.4</td>
<td>990.0</td>
<td>9900.0</td>
</tr>
<tr>
<td>L11</td>
<td>45.0</td>
<td>2319.6</td>
<td>1120.0</td>
<td>11200.0</td>
</tr>
<tr>
<td>L12</td>
<td>45.0</td>
<td>6213.2</td>
<td>3000.0</td>
<td>30000.0</td>
</tr>
<tr>
<td>L13</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A very important input for the characterization of unconventional shale reservoirs was the initial natural joint system orientation. Initial joint set orientations were derived from Reservoir outcrop fracture studies, core data, and log interpretation. In addition to the bedding plane, the model was initialized with three vertical joint sets:

- Joint set 1 dip direction 135°/dip magnitude 85°= set 1_outcrop study
- Joint set 2 dip direction 180°/dip magnitude 85°= set 2_outcrop study
A micro-seismic moment tensor analysis was used to verify shear plane orientations. Cluster analysis was performed using event locations from the seismic moment tensor analysis which passed the Quality Control Test. A total of 233 micro-seismic events with various cluster algorithms were used to identify the prominent joint set dip directions and dip magnitudes.

All tested cluster algorithms detected one reliable cluster:
- **mean value:**
  - dip direction: $50.5^\circ + 90^\circ = 145.5^\circ$
  - dip magnitude: $92^\circ$

The cluster analysis of micro-seismic events consistently indicated the activation and orientation of first vertical joint set.
3.4 Definition of Hydraulic Parameters

The fluid density and fluid viscosity were usually defined as an average fluid parameter for the entire stage. Because varying amount of gel additives were used during the frac jobs, an average effective fluid viscosity was calculated and applied to the model. This viscosity term affected the fracture system; not the wellbore. The projected bottom hole pressure during the frac job was applied as a boundary condition in the model, and this was provided by the service company performing the frac job.

Based on measurements used for the definition of the stress dependency of the fracture conductivity function, a limit stress without proppant placed for all joints (D1) of 4000 psi was defined, and a limit stress with proppant placed for all joints (D2) of 12000 psi. The minimum hydraulic conductivity scaling factor without proppant placed for all joints was 0.001 inches. The minimum hydraulic conductivity scaling factor with proppant placed was 0.01 inches for all joints. The shape factor of the stress dependent function was defined at 2.0 for all joints. These terms represented the deterioration of fracture conductivity with increasing normal stress subject to the defined limits.

Specific storativity was defined to be either constant or varying with time. From experience in other fields, a specific storativity value was defined, and then calibrated with the measured bottom hole pressure response of the frac job.

Hydraulic properties of the well bore conductivity were defined at 150 (ft/s), having a well bore diameter of 4.67 inch. The hydraulic properties of the perforation tunnels with a pressure drop of 300 psi at average slurry rate of 50 bpm resulted in a perforation tunnel conductivity of $K_{\text{perf}} = 0.31$ (ft/sec).

Based on measurement data, anisotropic initial horizontal and vertical permeability values were used. To estimate the effective drainage radius, the following empirical formula was applied:

$$R [\text{ft}] = 11.5 \sqrt{k_{\text{ini,}h} [nD]}$$  \hspace{1cm}\text{(3-1)}$$

where $k_{\text{ini,}h}$ is the initial horizontal permeability in the element (in the layer).

The drainage reservoir volume was calculated assuming an initial horizontal permeability of 10 nano Darcy for upper pay zone layers and 30 nano Darcy for lower pay zone layers. As a result drainage radii of 36 ft for the upper layers and of 73 ft for the lower layers was applied.

3.5 Initial in-situ Pore Pressure and Effective Stress Conditions

Initial pore pressure is defined for all layers using an initial pore pressure gradient of 0.74 psi/ft. Initial in-situ stress field is defined as effective stress for every layer of the reservoir by using a vertical total stress gradient (overburden gradient) of 1.08 psi/ft and conventional relationship between effective vertical stress $S_Z$ and effective minimum horizontal stress $S_{H,\text{min}}$ ($k0$-values) as well as effective maximum horizontal stress $S_{H,\text{max}}$.

Values for $k0$ for every layer vary between 0.4 and 0.8. The $S_{H,\text{max}}$ is defined to be an increment of 30% of the difference between $S_Z$ and $S_{H,\text{min}}$ relative to $S_{H,\text{min}}$. The direction of maximum horizontal stress direction was defined as being essentially perpendicular to the well direction.
4 Calibration
Because of the numerous uncertainties in reservoir conditions, calibration of the simulator to the frac job report and to microseismic events was important. A step wise calibration process was used which checked the plausibility and balance of simulator inputs to:

- ensure that in-situ strength, stress, and pore pressure values do not result in unrealistic plastic deformation
- ensure that the model starts and stops fracturing at DFIT/ISIP conditions
- ensure that the model represents fracture growth in time and space by matching the pressure and the pumping rate histories
- ensure that the volume balance between pumped fluid and created fracture volume results in the expected fluid efficiency
- ensure a reasonable match to microseismic measurement, that the model shows plausible fracture direction, extension, and fracture barriers.

During these plausibility controls and calibration steps, significant input parameters are fully checked and calibrated before starting a full systematic sensitivity analysis. The objective is to establish a set of parameters with a defined variation space that fulfills all verifications and gives the best possible fit respective of all available data and their associated uncertainties.

4.1 Calibrating of Fracture Start and Stop Conditions
After verifying that the in-situ initialization of the anisotropic stress field and the pore pressure conditions of the reservoir rock did not violate the material strength definition, the pressure levels where fractures start and stop were verified. Diagnostic Fracture Injection Test (DFIT) analysis and the pressure levels from ISIP (initial shut in pressure) define where fracture initiation and representative fracture extension pressures occur. There was also an estimation of the uncertainty of the data, with estimated minimal, mean, and maximal DFIT/ISIP conditions. After initializing a model with bottom-hole pressure to DFIT/ISIP pressure conditions, a model check was made to view the simulation results of the expected start and stop of fracture growth.

Typical adjustments during calibration to DFIT/ISIP conditions were:

- Adjustments of pore pressure and in-situ stress conditions at and around the perforation layer
- Adjustments of strength definition of the fracture mode (joint set) where the fracture starts and stops.

4.2 Calibration of Bottom Hole Pressure Response
After verifying the pressure levels associated with fracture starting and stopping, a verification of fracture growth rate was made. When pumping a frac stage, the resultant Bottom Hole Pressure (BHP) signal coupled with rate represents the speed at which the fracture network and fracture conductivity was created subject to the defined stress and strength conditions in the reservoir. The principal properties calibrated in this step included:

- the activated mean average joint distances in the different layers for intact rock failure and natural joints activation
- maximum effective hydraulic opening in the joint network
- strength properties of the reservoir rock outside perforation layer
• the overall loss in energy due to friction, leak off, turbulent flow or other dissipate mechanism that was summarized into the specific storativity value of the Darcy flow equation.

For calibration the initialization was made with a slurry rate history, with the resulting model BHP history measurements being compared with the reported BHP, cf. Fig. 4-1. A check was made of the volume of the resultant network of fractures in both simulations. Integration over all joints considering the mechanical joint openings was performed, and a comparison of the fracture volume to the fluid volume pumped was made. Assuming very low fluid leak off during fracing of unconventional reservoirs, a system in balance resulted in a fracture volume that should be equivalent to the total fluid volume pumped.

**Fig. 4-1**  
Stage 1, comparison between calculated BHP using SR Input (red) and BHP during pumping (blue)

### 4.3 Calibration of the Numerical Model to Microseismic Data

In addition to the bottom hole pressure signal, the model was calibrated with respect to the microseismic events reported during hydraulic fracturing. The microseismic events were measurements of micro “earthquakes” caused mostly by the shear failure of the jointed rock during the fracture process. The measurements provided the time, the position (point), and the magnitude of the individual events. The microseismic event catalog was used as a representation of the fracture network extension directionally. During the calibration of the model, the microseismic location was compared to the simulated plastic activity of fractures. The microseismic events and the plastic elements were plotted together at different time steps. This allowed a visual comparison of spatial distribution of both of the data sets. Fig. 4-2 shows the plot of plastic elements, the simulated connected proppant-accepting elements, and the microseismic events during the pumping of stage 1. The plot showed that the stimulated rock volume represented the fracture extension in terms of half-length and fracture height containment indicated by the microseismic events.
4.3.1 Use of Microseismic to Verify Barrier Modeling

Fracture barriers can generally be formed by elasticity, in-situ stress, strength horizons, or a combination of the three. In the beginning of the reservoir modeling, knowledge about the natural joints in every layer was limited. The model was first initialized with natural joint sets in every layer which potentially may have natural joints or planes of weakness. The calibration with microseismic data usually ensures quality information about fracture barriers, elasticity horizons, and in-situ stress horizons. These were checked to ensure sufficient strength to act as fracture barriers. On occasion, it was necessary to characterize a layer with no natural planes of weakness or joint sets although this is usually the exception, not the rule. In the Reservoir, layers 5 and 12 were modelled without any vertical natural joint sets. Using this approach, the microseismic results were modeled properly with respect to fracture height and fracture half-length. This initially required joint strength values for shear and tensile strength comparable to the intact rock strength values (Fig. 4-3).

**Fig. 4-2** Plot of connected proppant-accepting elements and microseismic events at the end of pumping

**Fig. 4-3**: Fracture grow with barriers having vertical joints strength close to intact rock strength values
4.4 Systematic Sensitivity Study to Reservoir Uncertainties

All previous calibration steps resulted in a model that demonstrated reasonable agreement with microseismic and pressure data. The model was then suitable to run systematic sensitivity analyses with respect to both the variation space of rock parameters as well as the variation space of design parameters such as well construction and fracture stimulation. Additionally, these variation windows were compared, and parameter influences were then be ranked.

An important point of interest was the check of the sensitivity of uncertainties to the barrier modeling. For the base model of the sensitivity study, the pre calibrated one stage model with a prescribed slurry rate boundary condition was used. For setting up the uncertainty windows of linear elastic parameters, the variation windows provided by the customer for horizontal and vertical Young’s moduli, Poisson’s ratio, and shear moduli per layer were used. An uncertainty window of UCS-values for every reservoir layer was available from core and log data. The uncertainty windows of tensile strength, friction angle and resulting cohesion values for the intact rock strength were developed. Uncertainty of joint set orientation was given from the natural fracture mapping of outcrops, logs, and cores in the reservoir to +/- 3° for bedding planes and +/- 10° for vertical fracture planes. Uncertainty windows for dilatancy angle, the strength definition of bedding planes and vertical joints, and the uncertainty of mean effective activated joint set distances for all layers were defined from experience with other similar fields. Uncertainty in anisotropic initial permeability values as well as initial pore pressure and initial stress conditions were given by measurement data. To investigate the sensitivity of barrier modeling to reservoir uncertainties, Layer 5 vertical joint set were defined having 80% of the strength of the intact rock of the layer.

Some of the uncertain parameters did not scatter independently. For example, the orientation of the natural fracture system was many times consistent throughout in all pay zone layers. After introducing all assumed correlations, the number of uncorrelated uncertainties was 158 variables.

4.4.1 How to Effectively Scan the Design Space in the Case of Having a Large Number of Variation Parameters

Sensitivity studies having a large number of uncertain parameter; particularly those having CPU intensive, nonlinear CAE-based design evaluations; calls for the most effective Design of Experiments (DOE) and correlation analysis strategy. That task was managed by using an optiSLang sensitivity workflow to generate a Metamodel of Optimal Prognosis (MOP) for every important response value. For this workflow, an Advanced Latin Hypercube sampling method controled selection of the input parameters respective of their variation space, and Coefficients of Prognosis (CoP) measurements verified the forecast quality of the resultant predictive MOPs [16].

After 200 design evaluations, the correlation structure of important input parameters showed convergence and the sensitivity analysis was stopped. Some designs did show unrealistically high plastic volume grow in single layers or did not converge in the initialization. These parameter configurations (design points) leading to failed simulations were sorted out. For this project, 117 designs out of the initial 200 designs were used for post processing.

4.4.2 Responses

To identify, check, and calibrate the dominant input parameters, the following response values were investigated:

- Total plastic volume, which is the total stimulated rock volume (SRV)
- Valuable Proppant-Accepting Stimulated Rock Volume (VSRV)
• Sum of element volumes having plastic activity at barrier layer (Layer 5)
• Fracture height, half-length, and fracture density function and the related shape error

### 4.4.3 Principal Results of the Sensitivity Analysis

After achieving a reasonable match to the Bottom Hole Pressure function, the main objective of the continued analysis was to improve the fit to the microseismic event catalog.

An investigation into the simulation results then focused on the relative penetration of the soft barrier layer 5. Out of the 117 viable design simulations, only 15 designs clearly matched the microseismic event observations with regard to this apparent frac barrier. This indicated that the strength values of layer 5 barrier (joint sets have 80% of intact rock strength) represented a much too small strength. As a result, all vertical planes of weakness were removed from layer 5. In addition, correlation analysis of the plastic activity in the barrier, cf. *Fig. 4-4*, showed the importance of strength parameters of the intact rock in the barrier (layer5\_Phi= friction angle, layer5\_Rd= UCS values), of in-situ stress (layer5\_k0) and horizontal Young’s modulus (layer5\_Eh). As a result, the UCS values was set to a higher value, along with friction angle and k0 which were further calibrated to support the premise of loading and penetrating the barrier only via tensile failure of intact rock devoid of any natural fracture system.

![Graph showing Coefficient of Prognosis](image)

*Fig. 4-4: Coefficient of Prognosis plot for the response value SumOfVertJointActivity (volume of elements with vertical joint set activity in layer) which shows the most important uncertain reservoir parameter to the variation of the response*

To measure the improvement of fit to the microseismic event catalog, a density function approach was used. A normalized density function of the microseismic event cloud was compared with a normalized density function of plastic strain. Using an objective function, important reservoir values were modified to obtain a better fit to the microseismic data.

- UCS\_5 intact compression strength = 42500 psi
- Phi\_5 intact friction angle = 45°, with related tensile strength
- K0\_5 = 0.522

Additional simulation runs were then obtained. By modifying three important reservoir parameters and removing initial vertical joints in the barrier, a much better fit to microseismic data was obtained.
5 Sensitivity to Operational Conditions

Sensitivity analyses relative to the operational parameters was then performed in order to establish the cost/benefit relationship related to well drilling and completion practices. EUR improvements were derived from increases in the valuable stimulated rock volume (VSRV) and the resulting increase in accessible gas initially in place (AHCIIIP).

5.1 Reference design for sensitivity analysis

As reference design, the calibrated model of hydraulic fracturing was used. In order to forecast variation of valuable stimulated rock volume and drainage volumes, multiple stage simulations were required. The initial simulation stage did not see stress shadowing, but the effect of stress shadowing is an important consideration for the optimization of the stage design. A multistage simulation exercise is required to better capture the stress caging or stress shadowing effects. Experience has shown that three stages was adequate.

5.2 Parameterization

In the sensitivity analyses, the following parameters representing different operational conditions are varied:

- well depth
- definition of perforation and stage design (stage length, number of perforation clusters and stage spacing)
- pumping regime (slurry volume and slurry rate signals)
- average slurry viscosity

The number of perforations and the possible well depths were defined as discrete parameters. All other parameters vary between lower and upper bounds. To be able to modify slurry rate and total slurry volume using a parametric procedure, the slurry rate function was idealized to be identical for every stage and having identical waiting time between stages.

5.3 Scan of the Design Space

Using and optiSLang workflow of sensitivity analysis [4], after 76 design evaluations, the correlation structure of important operation parameters showed convergence in the solution, and the COP values of the important MOPs were large enough to imply confidence in the Metamodel predictability. Consequently, the sensitivity analysis was stopped.

5.4 Responses to Evaluate

The influence of the variation of the operational parameters was then quantified by the measurement of valuable stimulated rock volume (VSRV) and the related accessible hydrocarbons initially in place (AHCIIIP). Volumes having joint set openings > 3 times mean proppant size = 1.0 mm, were presumed to have accepted proppant. The VSRV was calculated as the total connected proppant-accepting volume by selecting only elements having proppant-accepting opening with a viable connection path to one of the perforation clusters.

In addition, the VSRV was calculated based on the drainage volume and hydrocarbon content. The calculation of the accessible gas initially in place (AHCIIIP) was then performed as follows:
\[ \text{AHCIIIP} = \sum_{i=1}^{n_L} V_{\text{drain},i} \cdot V_{g,sfc,i} \]  \hspace{1cm} (5.1)

where \( n_L \) is the number of layers, \( V_{\text{drain},i} \) was the drainage volume of the \( i \)-th layer and \( V_{g,sfc,i} \) [v/vbulk] was the volume of gas at surface conditions stored in one cubic foot of formation in the \( i \)-th layer.

To investigate the importance of stage design on the final AHCIIIP, the accessible hydrocarbons initially in place (AHCIIIP\(_{\text{well}}\)) were calculated over the total length of a horizontal well:

\[ \text{AHCIIIP}_{\text{well}} = (\text{AHCIIIP}_{\text{stages}3} - \text{AHCIIIP}_{\text{stages1&2}}) \cdot \left( \frac{l_{\text{well,tot}}}{\Delta_{\text{Stage}} + l_{\text{Stage}}} - 2 \right) + \text{AHCIIIP}_{\text{stages1&2}}, \]  \hspace{1cm} (5.2)

where \( l_{\text{well,tot}} \) was the total horizontal well length, \( \Delta_{\text{Stage}} \) was the stage spacing and \( l_{\text{Stage}} \) was the stage length. Note that AHCIIIP\(_{\text{well}}\) separate the virgin stages 1 and 2 and assumed a repeatable performance of the additional AHCIIIP for all additional stages.

### 5.5 Results of Sensitivity analysis

For this project, the sensitivity to the connected proppant-accepting volume (VSRV) after three stages of hydraulic fracturing showed the dominance of slurry volume and of fluid viscosity, see Fig. 5-1.

![Plots of Coefficient of Prognosis (CoP) and related Meta model of optimal Prognosis (MoP) for VSRV after 3 stages](image)

To envision the importance of stage design (length, clusters), it was necessary to extend the concept of VSRV. Multiplying VSRV with the gas content in the drainage reservoir volume for each layer results in the AHCIIIP. The relative importance of stage design (stage spacing) appears, see Fig. 5-2.
Using the measures of $\text{AHCIIIP}_{\text{well}}$, cf. Fig. 5-3, the number of perforation appears as an important operational parameter. The objective value was sensitive to:

- The stress shadowing effect between the stages which will have a tendency to reduce the incremental $\text{AHCIIIP}_{\text{well}}$ as the stages are located closer to each other
- The effect of losing of potential $\text{AHCIIIP}_{\text{well}}$ by having too much unstimulated rock along the horizontal well as a result of placing the perforation clusters too far from each other

The optimizer balanced between these two contradictory effects, and selected optimal distances between the perforation clusters/stages. The best possible compromise was also influenced by the effective drainage radius.
5.6 Optimization of AHCIIIP Well

For the Fracture Stimulation optimization phase, the MetaModel derived from the many sensitivity runs was used. The MOP mathematical correlation model captured the correlations between the variation of the operational parameters and the variation of the responses. Having CoPs ranging from 70% to 90%, these MOP functions explained the majority of the response variation; essentially how variation of the completion parameters influenced the hydrocarbon production. With a strong CoP value, these correlations were credible for use in optimization.

![Evolutionary Algorithm](image)

**Fig. 5-4 optimization flow in optiSLang**

The MOPs derived principal response values (AHCIIIP) at the end of stage 2 and 3 were used to calculate the objective AHCIIIPwell for every design which was evaluated during the optimization.

![Optimization Table](image)

**Fig. 5-5 Definition of objective to me maximized in optiSLang**

For this model, the variation of landing depth in the sensitivity analysis did not show any significant relevance due to the responses in the investigated range of variation. With a constraint of fracture height between the two fracture barriers layer 5 and 12, the variation did not result in significant differences between VSRV and AGIIIP values. As a result, the landing depth was not a parameter of the optimization process.
Using the MOPs, the following values finally were optimized:

- cluster per stage
- stage length
- distance between stages
- slurry rate
- slurry volume
- fluid viscosity

The potential gains with these optimizations appeared significant at virtually the same completion cost. The potential for doubling the AGIIP\textsubscript{Well} (proxy for EUR) was indicated.

5.7 Outlook how to balance between additional completion costs and EUR

Experience shows that the opportunity to significantly improve HCIIP\textsubscript{SRV,Well} and AHCIIIP\textsubscript{Well} is limited if the available completion costs are fixed. There appears to be a significant opportunity to potential to improve with inventions in completion. But we need to check that the increase of completion cost’s of hydraulic fracturing will not overprint the benefit from optimization of accessible HCIIP.

Of course that only occur if the additional costs for well completion are not covered by the additional oil and gas production. To evaluate the possible conflict, the optimization needs to be enlarged due to cost function as well as revenue function out of the additional hydrocarbons. In order to do so we need revenue estimations for additional oil and gas production as well as completion costs (UDC) estimations for the additional stage designs and slurry volume.

To use suitable correlation between AHCIIIP\textsubscript{Well} and estimated ultimate recovery (EUR) it is recommended to calculate multiple wells nearby the calibration well using the MOP to calculate AHCIIIP\textsubscript{Well} and extract a correlation between AHCIIIP\textsubscript{Well} and estimated EUR of the wells.

Having cost functions in place optiSlang can be used to calculate the Pareto Frontier, which represents all completion designs were additional improvement of hydrocarbon production can only be achieved with additional investments in completion. Fig. 5-6 shows a demo Pareto Frontier calculated with the help of the MOP using anonymised correlations between AHCIIIP\textsubscript{Well}, EUR and dummy cost functions. Having such kind of Pareto Frontier decisions can be made regarding operational parameters in relation to increase of completion costs and EUR at the same time. To deal with the Pareto Frontier as an example three base scenarios are possible:

- optimize EUR to the maximal values and check completion costs
- what is the best possible EUR without increase of current completion costs
- or minimize completion costs keeping current EUR of current
- completion scenario
Fig. 5-6 Pareto Frontier showing the conflict between increase in EUR and increase in completion costs as well as the three base scenarios

6 Summary

Using the Dynardo simulator, a fully three-dimensional fracture simulation model respective of reservoir and completion data was built and calibrated to the best available sets of core, log, DFIT/ISIP, fracture completion, and microseismic data. The integrated workflow approach was able to deliver a predictive model applicable to the calibration well and to neighboring wells.

Based on a calibrated parametric reservoir model and sensitivity analysis to operational parameters, the hydraulic fracturing design was modified. The completion recommendations developed by this work were implemented in the field, and well performance improvement with respect to EUR performance was improved consistent with Dynardo model predictions.

References


