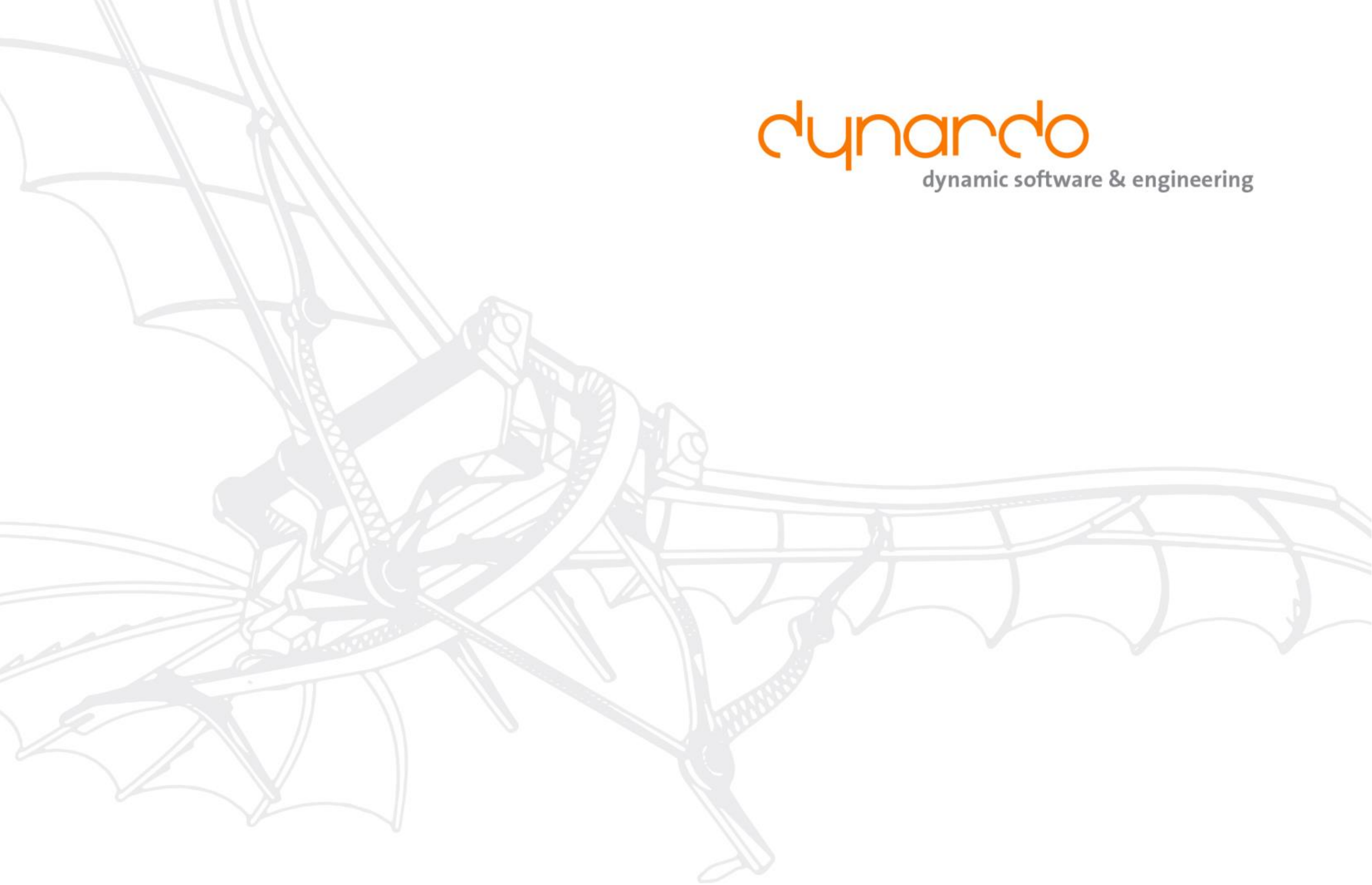

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Risk Analysis - Assessment of Reliability for Concrete Dams

M. Goldgruber, R. Lampert, D. Hinterdorfer

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Risk Analysis - Assessment of Reliability for Concrete Dams

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ABSTRACT: The stochastic analysis of the dam monolith in this benchmark workshop shows the applicability of the method for dam engineers. This method has the advantage that no partial factors of safety must be applied on the action or resistance side. On the other hand, applying stochastic methods to limit state equation is indeed convenient, but talking about to more complex systems, like nonlinear finite element with 1000000 degrees of freedom, the analysis of only one limit state might take a week. For the automatized simulation process, the evaluation of failure criteria and to keep a set of hundreds of simulations manageable, a tool like ANSYS *optiSlang*® is essential for such complex analyses. However, also in this relative simple analysis, the tool ANSYS *optiSlang*® provided a convenient way to define parameters and evaluate failure probabilities and reliability indexes automatically for all design situations. The results show that the sliding safety of the dam monolith, by means of the target reliability index β_T, isn't achieved, neither for sliding at the dam-rock interface nor at the rock-rock interface. Additionally to the popular FORM method used in this benchmark example, Monte Carlo analyses of the same problem are performed, which yield a very good agreement between these two methods. The Swedish "Probabilistic model code for concrete dams" delivers a comprehensive, but easy to use document for such analyses.

1 Introduction and problem description

The problem description [1] provided by the formulators includes the data for the dam monolith [2] and the Swedish probabilistic model code for concrete dams [2]. Limit states and design situations as well as statistical distributions are described in the model code [2]. Reading instructions are included at the first page of the appendix. Statistical distributions related to flood event is given in the dam data sheet [2].
The object of interest is a concrete dam monolith in northern Sweden. The dam has a total height of 25 m and a width of 12 m. There is a inspect gallery located at the upstream dam heel, but no drainage or grout curtain is situated in the foundation to reduce the pore water pressure, and therefore the uplift. The total volume of the dam is 2870 m^3. Reducing the dimension to 2D gives a total area of the dam section of 239.2 m^2. To increase the resistance to overturning and sliding an anchor is situated near the upstream surface of the dam, with a post-stressed total force of 1080 kN/m. The long-term loss of this force is estimated to 10 %. Apart from the possible sliding case of such a structure between the dam-rock interface there is also a possibility of sliding along rock-rock interface due to a rock-joint of 20° at the dam's foundation. The volume of this granite rock wedge is estimated to be 1376 m^3, hence in the 2D case 114.7 m^2. Additionally to the hydrostatic water load a second horizontal action comes from the ice load. The downstream water level isn't taken into account in safety assessment. Also, the reduced weight of the dam monolith due to entrance to the inspection gallery (Section B-B, Figure 1) is neglected and assumed to be fully filled with concrete. Figure 1 illustrates the dimensions of the dam monolith.

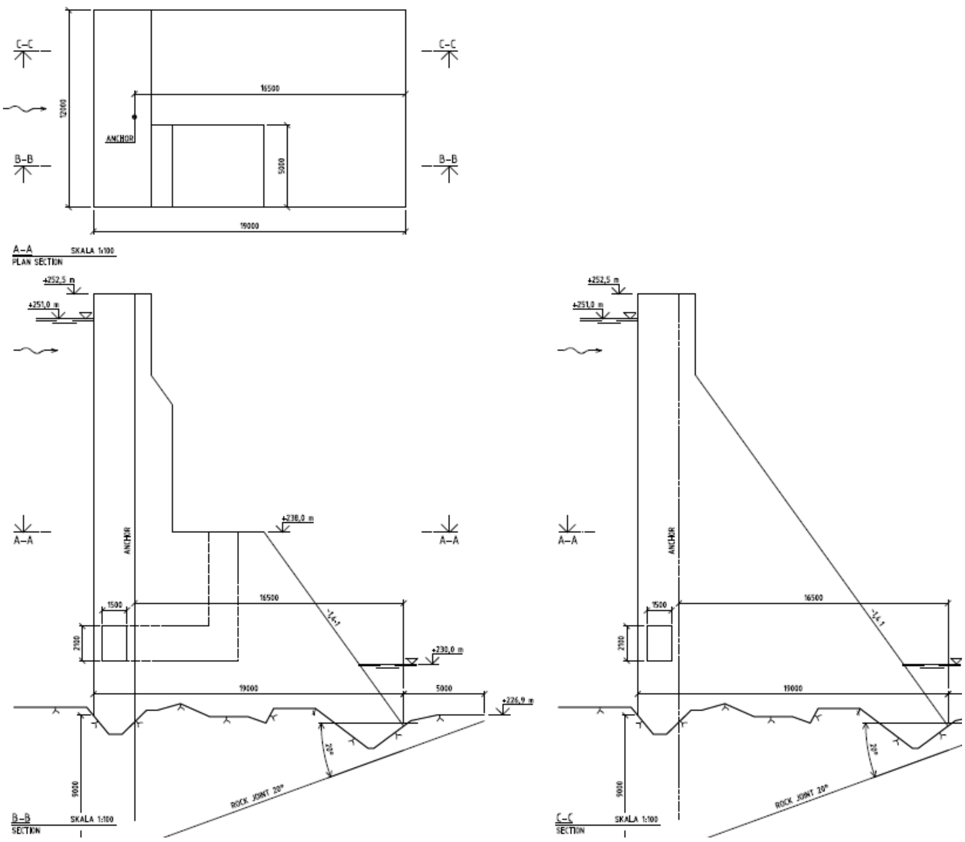

Figure 1: Dam monolith [2].

The formulator of this theme stated the following tasks in [1] to be solved:

1. *Estimate the deterministic factor of safety for sliding considering 2 failure modes:*
   - *(a) sliding along the dam-foundation contact, and*
   - *(b) sliding along an existing joint in the foundation.*

   For each failure mode, the factor of safety shall be calculated in 2 situations:
   - *(i) normal load case, and*
   - *(ii) (ii) flood load case. In the normal load case water is at retention water level (rwl), the dam is subject to an ice load of 200 kN/m acting 1/3 m below rwl. In the flood load case water is at the dam crest level, with no ice load present. Uplift in the concrete-rock contact is assumed according to standard procedure, with no reduction since there are no drain holes. In the rock joint a linear pressure distribution from reservoir level to downstream level is assumed.*
2. *Define limit state functions for the 2 failure modes considered: (a) sliding along the concrete-rock contact and (b) sliding along a rock joint in the foundation.*
3. *Estimate the probability of failure* for the 2 failure modes considered for i) a normal design situation and ii) for an exceptional design situation (flood). In total, 4 probabilities of failure have to be provided.*
4. *Present sensitivity values** for all 4 cases.*
5. *Estimate the system reliability of the monolith (for both limit states and both design situations).*
6. *Consider that two additional shear tests are performed on the concrete/rock contact to determine the basic friction angle. How does this change the failure probability of the normal design situation for sliding along the concrete-rock contact?*

2 Deterministic factors of safety

The first part of this theme is to estimate/calculate the deterministic safety of sliding according to the limit equilibrium approach. All forces acting on the dam are depicted in Figure 2. Note that due to the possible sliding failure along the rock-joint, the anchor force A cannot be taken into account anymore and the hydrostatic water pressure is assumed to be acting to the depth of the rock wedge on the upstream side. Furthermore, the uplift force U is now acting along the joint and oriented normal to the interface. For the flood load case, no ice pressure is taken into account.

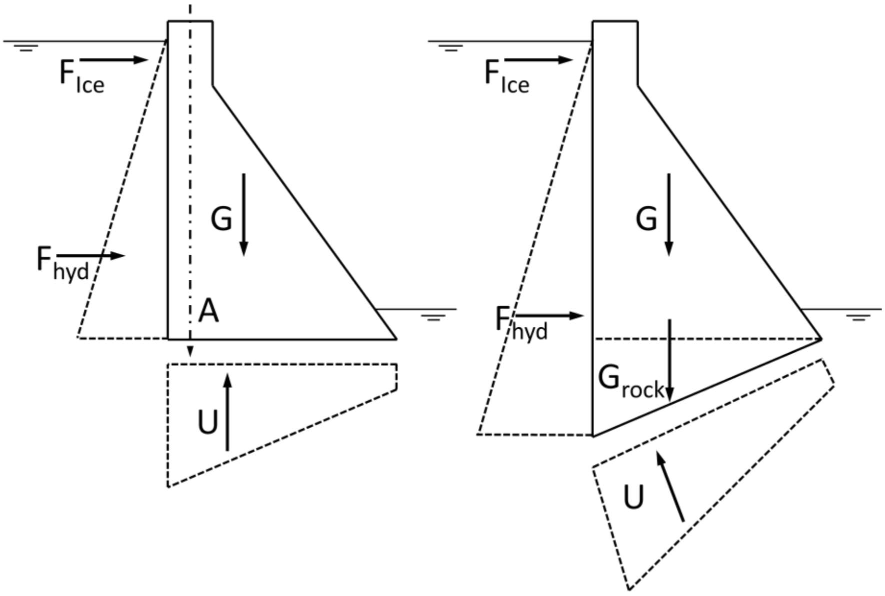

Figure 2: Forces acting on the dam.

In total 4 different cases have to investigated:

1. Normal load case NLC (retention water level)
   a. Sliding at the dam-rock interface

$$Resistance_{NLC,d-r} = (G_d - U_d + A)\ tan(phi + i)$$

$$Action_{NLC,d-r} = F_{hyd,rwl} + F_{Ice}$$

   b. Sliding at the rock-rock interface

$$Resistance_{NLC,r-r} = (G_{d,r}\ cos(\alpha) + F_{hyd,rwl,r} sin(\alpha) + F_{Ice} sin(\alpha) - U_r)\ tan(phi + i)$$

$$Action_{NLC,r-r} = -G_{d,r}\ sin(\alpha) + F_{hyd,rwl,r} cos(\alpha) + F_{Ice} cos(\alpha)$$

2. Flood load case FLC (flood water level)
   a. Sliding at the dam-rock interface

$$Resistance_{FLC,d-r} = (G_d - U_d + A)\ tan(phi + i)$$

$$Action_{FLC,d-r} = F_{hyd,Flood}$$

   b. Sliding at the rock-rock interface

$$Resistance_{FLC,r-r} = (G_{d,r}\ cos(\alpha) + F_{hyd,Flood,r} sin(\alpha) - U_r)\ tan(phi + i)$$

$$Action_{FLC,r-r} = -G_{d,r}\ sin(\alpha) + F_{hyd,Flood,r} cos(\alpha)$$

With

$$G_d = A_d\,\gamma_c$$
$$G_r = A_r\,\gamma_r$$
$$G_{d,r} = A_d\,\gamma_c + A_r\,\gamma_r$$
$$F_{hyd,rwl} = \frac{\gamma_w (h_{rwl})^2}{2}$$
$$F_{hyd,Flood} = \frac{\gamma_w (h_{Flood})^2}{2}$$
$$F_{hyd,rwl,r} = \frac{\gamma_w (h_{rwl} + h_r)^2}{2}$$
$$F_{hyd,Flood,r} = \frac{\gamma_w (h_{Flood} + h_r)^2}{2}$$
$$U_d = \gamma_w b_d h_{ds} + \frac{\gamma_w (h_{rwl} - h_{ds}) b_d}{2}$$
$$U_r = \gamma_w b_r h_{ds} + \frac{\gamma_w (h_{Flood} - h_{ds}) b_r}{2}$$

For the deterministic approach, the ice pressure F_{Ice} is assumed to be 200 kN/m.
The following Table 1 summarizes all necessary parameters for the deterministic safety assessment of the dam.

Table 1: Dimensions and specific weight.

Area of the dam A_d	239.2 m²
Specific weight of the concrete γ_c	23.5 kN/m³
Area of the rock wedge A_r	114.7 m²
Specific weight of the granite γ_r	25.0 kN/m³
Specific weight of water γ_w	10.0 kN/m³
Retention water level h_{rwl}	24.1 m
Depth of the rock wedge at the upstream heel h_r	9.0 m
Water level a flood h_{Flood}	27.6 m
Downstream water level h_{ds}	3.1 m
Dam-rock interface width b_d	19.0 m
Rock-rock interface width b_r	25.6 m
Rock-rock interface angle α	20°
Friction angle at the dam-rock interface	35°
Dilatation angle at the dam-rock interface	15°
Friction angle at the rock-rock interface	32°
Dilatation angle at the rock-rock interface	8°

For the normal load case NLC all forces are summarized in Table 2.

Table 2: Normal load case forces acting on the dam.

Normal Load Case		
Force [kN/m]	**Sliding at the dam-rock interface**	**Sliding at the rock-rock interface**
Vertical Forces:		
G_d	5620	5620
G_r	0	2868
U	-2584	-4639
A	972	0

Horizontal Forces:		
$F_{hyd,rwl,d}$; $F_{hyd,rwl,r}$	2904	5478
F_{Ice}	200	200
Actions	**3104**	**2480** **(Tangential to the Rock-Rock Interface)**
Resistance	**4777**	**4444** **(Tangential to the Rock-Rock Interface)**
Safety Factor = $\frac{\boldsymbol{Resistance}}{\boldsymbol{Actions}}$	**1.54**	**1.79**

For the flood load case FLC all forces are summarized in Table 3.

Table 3: Flood load case forces acting on the dam.

Flood Load Case		
Force [kN/m]	**Sliding at the dam-rock interface**	**Sliding at the rock-rock interface**
Horizontal Forces:		
G_d	5620	5620
G_r	0	2868
U	-2584	-4639
A	972	0
Vertical Forces:		
$F_{hyd,Flood,d}$; $F_{hyd,Flood,r}$	3809	6698
F_{Ice}	0	0
Actions	**3809**	**3391** **(Tangential to the Rock-Rock Interface)**
Resistance	**4777**	**4722** **(Tangential to the Rock-Rock Interface)**
Safety Factor = $\frac{\boldsymbol{Resistance}}{\boldsymbol{Actions}}$	**1.25**	**1.39**

The results of the safety factors show that sliding along the rock-rock interface is safer than at the dam-rock interface. Also, as one would expect, the flood load case safety factors are lower than at retention water level, although no ice load is taken into account at the flood event.

3 Stochastic analysis

In general, a stochastic analysis can be used to circumvent contradictions arising from the use of partial safety factors. Another issue is the question of the safety level of an existing building. It is possible in principle to provide a proof of stability by means of a concept based on safety factors. However, if an existing safety level is to be predicted on this basis until the building fails, the question arises as to whether it should be determined by a load-side increase or by a reduction of the resistance.

By means of a stochastic analysis, failure probabilities can also be determined by introducing load and resistance-side scatterings. Therefore, the stochastic analysis consists of the following steps:

- Definition of the scattering of the input parameters:
  For this purpose, distribution functions, mean values, coefficients of variation for all parameters used in the limit state functions are obtained from the "Probabilistic model code for concrete dams" [2].
- Defining limit State functions
  - g ≥ Resistance –Actions
- Evaluation of 4 different failure probabilities for sliding along each interface with the First Order Reliability Method (FORM):
  - Normal load case (Retention water level)
  - Water above rwl: 0.0<de<1.5
  - Water above rwl: 1.5<de<2.5
  - Water above rwl: 2.5<de

  The evaluation and determination of the probability of failure is carried out with ANSYS *optiSLang*® [4], various results and output options are available for the evaluation of a stochastic analysis.

In addition to the calculation of the probability of failure, the influencing parameters which are decisive for the distribution of the response variable can be output both qualitatively and quantitatively in ANSYS *optiSLang*® [4]. This allows statements to be made as to which stray input variables (loads, resistances) are relevant for the failure of the dam.

3.1 Statistical data of the input parameters

The following table summarizes all parameters used in the limit state functions in section 2, including distribution functions, mean values, standard deviation and coefficients of variation.

Table 4: Parameters, distribution functions, mean values, standard deviation and coefficients of variation.

Parameter	Distribution function	Mean value	Standard deviation	CoV
Specific weight of the concrete γ_c [kN/m³]	Normal	23.50	0.94	0.04
Specific weight of the rock γ_r [kN/m³]	Normal	25.88	0.23	0.01
Uplift parameter C [-]	Normal	1.00	0.05	0.05
Anchor force P_0 [kN/m]	Normal	1080.00	81.00	0.08
Anchor force loss dP [kN/m]	Normal	108.00	32.40	0.30
Ice load F_{ice} [kN/m]	Lognormal	80.00	80.00	1.00
Max. ice load Ice_{max} [kN/m]	Normal	250.00	25.00	0.10
Friction angle φ [°]	Normal	35.00	1.75	0.05
Dilation angle i [°]	Lognormal	15.00	3.00	0.20
$d_{e,0.0-1.5}$ [m]	Triangular	1.03	0.77	0.74
$d_{e,1.5-2.5}$ [m]	Triangular	1.13	0.67	0.59
$d_{e,>2.5}$ [m]	Triangular	0.90	0.84	0.94

Additional conditions have been added to the ice load, because according to [3] the maximum ice load from F_{ice} must not exceed the maximum Ice_{max}. The retention water level h_{rwl} is set constant, but the heights of exceedance d_e have triangular distributions, with truncations at the specific boundaries they are defined for. Specific values for their triangular distribution can be found in Table 5 and the cumulative distribution function (CDF) of these three parts in Figure 3.

Table 5: Triangular distribution parameters.

	Part 1 **$0<d_e<1.5$**	**Part 2** **$1.5<d_e<2.5$**	**Part 3** **$d_e>2.5$**
A	-0.10	-0.10	-1.00
B	3.20	3.00	3.10
C	0.00	0.50	0.60
Min [m]	0.00	1.50	2.50
Max [m]	1.50	2.50	3.50

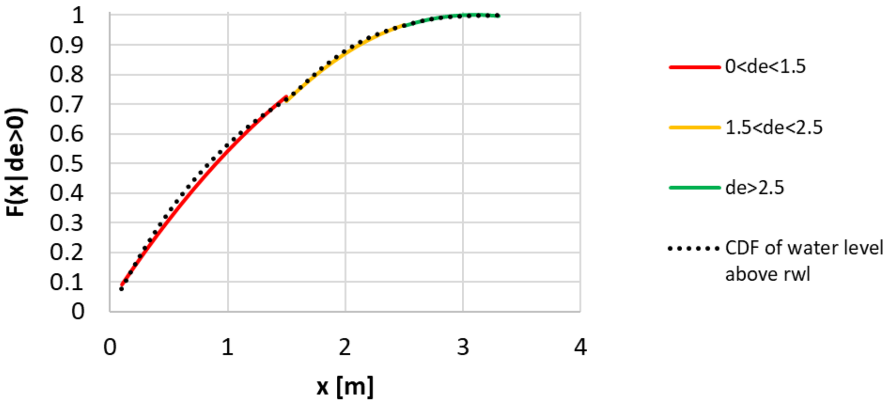

Figure 3: CDF of the water level divided into 3 parts.

The probability of exceeding the retention water level is given in [2] and is $P(d_e>0) = 3.0E\text{-}3$. Hence, the probabilities for exceeding the maxima of each part can be calculated according to Table 6.

Table 6: Probabilities for exceeding the maxima of the 3 parts.

$P(d_e>0) =$	3.00E-03
$P(d_e>1.5) = (1-F(x=1.5 \vert d_e>0)) * P(d_e>0) = (1-0.716) * 0.003 =$	8.52E-04
$P(d_e>2.5) = (1-F(x=2.5 \vert d_e>0)) * P(d_e>0) = (1-0.965) * 0.003 =$	1.05E-04
$P(0<d_e<1.5) = P(d_e>0)-P(d_e>1.5) =$	**2.15E-03**
$P(1.5<d_e<2.5) = P(d_e>1.5)-P(d_e>2.5) =$	**7.47E-04**
$P(d_e>2.5) =$	**1.05E-04**

3.2 Results

Table 7 and Table 8 are summarizing the probabilities of failure P_f and reliability indexes β. Design situation 1 and 4 indicate the normal loading case and the flood loading case, respectively, according to “Table PI-7-1. Design Situations” from [3]. In the flood loading case, the conditional probability of the event must be considered, therefore, in the flood load case the failure probability of the persistent situation 4.0 (rwl, without ice) must also be taken into account. Design situations 4.1 to 4.3 are the three depths, respectively. The sum of 4.0 to 4.3, hence corresponds to the cumulative failure probability of the flood event.

Table 7: Sliding failure probability and reliability index of the dam monolith at the dam-rock interface.

		Design Situation						
		1 **rwl=24.1m**		**4.0** **d_e=0**	**4.1** **0<d_e<1.5**	**4.2** **1.5<d_e<2.5**	**4.3** **d_e>2.5**	**4** **d_e>0**
Probability of Occurence	P_{occur}	1.00E+00		1.00E+00	2.15E-03	7.47E-04	1.05E-04	
Respective Probability of Failure	P_{rf}	1.10E-04		4.31E-05	1.44E-03	1.76E-03	1.13E-02	
Failure Probability **Prob. of Failure * Prob. of Occur.**	P_f	**1.10E-04**		**4.31E-05**	**3.09E-06**	**1.45E-06**	**3.36E-07**	**4.87E-05**
Reliability Index	**β**	**3.70**		**3.93**	**2.98**	**2.92**	**2.28**	**3.90**

Table 8: Sliding failure probability and reliability index of the dam monolith at the rock-rock interface.

		Design Situation						
		1 **rwl=24.1m**		**4.0** **d_e=0**	**4.1** **0<d_e<1.5**	**4.2** **1.5<d_e<2.5**	**4.3** **d_e>2.5**	**4** **d_e>0**
Probability of Occurence	P_{occur}	1.00E+00		1.00E+00	2.15E-03	7.47E-04	1.05E-04	
Respective Probability of Failure	P_{rf}	3.83E-06		1.83E-06	1.15E-04	1.40E-04	1.27E-03	
Failure Probability **Prob. of Failure * Prob. of Occur.**	P_f	**3.83E-06**		**1.83E-06**	**2.47E-07**	**1.15E-07**	**3.79E-08**	**2.31E-06**
Reliability Index	**β**	**4.47**		**4.63**	**3.68**	**3.63**	**3.02**	**4.58**

Due to the fact, that no dam consequence class or target safety index is given by the formulators, consequence class B is assumed, see [3], which may cause loss of human lives and failure may lead to large regional and local consequences and disturbances. Therefore, the minimum target reliability index, which should be achieved, is $\beta_T = 4.8$. Based on this target the dam monolith wouldn't be save, neither for sliding at the dam-rock interface nor at the rock-rock interface. It should be mentioned that for the persistent situation at retention water level the reliability index is the lowest, which means that failure probability is the highest. The respective failure probability of the dam at water levels above retention water level is in fact higher, but the probability of occurrence of such an event is much smaller, hence the probability to fail due to sliding is decreasing. Note, that not even for lowest consequence class D, see [3], with a target reliability index of $\beta_T = 3.8$ the reliability wouldn't be fulfilled.

3.3 Sensitivity values

The sensitivity values, by means of Coefficients of Prognosis (COP), are summarized in Table 9. The CoP was first introduced by Most and Will in 2008 [6], which is a model independent measure to assess the model quality. These values are a direct result and printout of the simulations in ANSYS *optiSlang*®, which give an excellent measure of the most important parameters of a reliability/sensitivity analysis. In each case the most important parameter is the dilation angle and the second one the friction angle. In the case of sliding at the dam-rock interface the influence of the specific weight of the concrete is also quite high.

Table 9: Coefficients of Prognosis (COP) for both design situations and sliding interfaces.

	Coefficients of Prognosis (COP) [%]			
	Dam-Rock Interface		Rock-Rock Interface	
Parameter	DS 1	DS4	DS1	DS4
γ_c [kN/m³]	15.38	16.35	14.80	10.26
γ_r [kN/m³]	0.00	0.00	0.00	0.00
C [-] (uplift parameter)	5.19	4.28	5.59	5.85
P_0 [kN/m]	1.46	0.74	0.00	0.00
dP [kN/m]	0.00	0.00	0.00	0.00
F_{ice} [kN/m]	0.00	0.00	0.00	0.00
Ice_{max} [kN/m]	0.00	0.00	0.00	0.00
φ [°]	18.67	17.54	32.01	33.26
i [°]	60.37	57.67	47.03	47.22

3.4 Failure probability of the normal design situation for sliding at the dam-rock interface due to additional shear tests

Two additional shear tests at the dam-rock interface are provided to evaluate the influence of a changing friction angle at the base of the dam on the probability of failure. The measured friction angles are 37° and 38°, hence the mean value is m=37.5°. The expected variation in the mean value between different dams according to [2] can be assumed normal distributed with a mean value of E(μ’) = 35°, a standard deviation of σ(μ’) = 1.75° and a variance of Var(μ’) = σ(μ’)² = 3.06°°. The on-site variability of the friction angle may be expected to have a coefficient of variation of 0.03, which gives σ = m * 0.03 = 1.125°.
The expected mean value and standard deviation due to the new measurements can be calculated according to [3] by

$$E_{(\mu'')} = \frac{m \cdot Var(\mu') + E_{(\mu')}\frac{\sigma^2}{n}}{Var(\mu') + \frac{\sigma^2}{n}} = 37.07° \qquad \sigma_{(\mu'')} = \sqrt{Var(\mu'')} = \sqrt{\frac{Var(\mu') \cdot \frac{\sigma^2}{n}}{Var(\mu') + \frac{\sigma^2}{n}}} = 0.724°$$

with the updated coefficient of variation

$$V_{\overline{\varphi}_b} \approx \sqrt{V_{\varphi_b}^2 + V_{stat,\overline{\varphi}_b}^2} \approx \sqrt{\left(\frac{E_{(\mu')} * 0.03}{E_{(\mu'')}}\right)^2 + \left(\frac{\sqrt{Var(\mu'')}}{E_{(\mu'')}}\right)^2} \approx \sqrt{\left(\frac{1.05}{37.1}\right)^2 + \left(\frac{0.72}{37.1}\right)^2} \approx 0.034$$

Applying these new values to the stochastic analysis of the design situation 1 for sliding at the dam-rock interface gives a new probability of failure of P_f = 3.76E-06 and corresponding reliability index of β = 4.48. Comparing these values with them from the analysis with the initial friction angles shows the wide influence of this parameter for the analysis of the sliding safety of the dam monolith. An increase of only 2° and a decrease of the standard deviation by 0.5° boosts the reliability index from β = 3.70 to β = 4.48, which is a huge improvement, because it reduces the failure probability P_f by a factor of 30. In this analysis the sensitivity values, by means of Coefficients of Prognosis have changed. The second most important parameter from the former analysis, the friction angle (COP approx. 30%), is moved back to be one of the less important ones (COP = 11%), because of the reduced variance and the increased mean value.

3.5 Additional Monte Carlo analysis of design situation 1 for the dam-rock interface

Additionally to the FORM method, Monte Carlo analysis are performed to validate the results. Therefore, the limit state for design situation 1 for sliding at the dam-rock interface has been used. The analysis is done with the program slangTNG [5]. The advantage of the tool is its convenient scripting possibilities and large library of stochastic and mathematical tools. The performed Monte Carlo analysis with 1,000,000 samples was done in less than a minute.

The failure probability calculated by doing a couple of Monte Carlo analyses lies between $0.00010 < P_f < 0.00012$, corresponds to a reliability index of approximately $\beta = 3.70$. This is practically the same as calculated by the FORM method. Additionally, Monte Carlo analyses (10,000,000 samples) for the adjusted friction angle from section 3.4 resulted in values of approx. $P_f = 3.0E\text{-}06$ and $\beta = 4.6$, which are also close to the FORM method ($P_f = 3.76E\text{-}06$; $\beta = 4.48$).

Hence, the FORM method yields reliable results, with the advantage of being much faster at more complex systems, e.g. finite element analysis. Nevertheless, the Monte Carlo analysis is only very practicable as long as the computation time of the problem is relatively short.

4 Conclusion

The stochastic analysis of the dam monolith in this benchmark workshop shows the easy applicability of the method for dam engineers if one has the right tools at hand and at least a basic knowledge in statistics and stochastic analysis. Furthermore, it has the advantage that no partial factors of safety must be applied on the actions or resistance side. On the other hand, applying stochastic methods to limit state equation is indeed convenient, but talking about to more complex systems, like nonlinear finite element with 1000000 degrees of freedom, the analysis of only one limit state might take a week. Based on this fact, a lot of computation power is necessary to simulate hundreds of such problems in parallel. For the automatized simulation process, the evaluation of failure criteria and to keep a set of hundreds of simulations manageable, a tool like ANSYS *optiSlang*® is essential for such complex analyses. However, also in this relative simple analysis the tool ANSYS *optiSlang*® provided a convenient way to define parameters and evaluate failure probabilities and reliability indexes automatically for all design situations. The results show that the sliding safety of the dam monolith, by means of the target reliability index $\beta_T = 4.8$, isn't achieved, neither for sliding at the dam-rock interface nor at the rock-rock interface. However, in this benchmark example the failure probability of sliding at the rock-rock interface is more unlikely than sliding at the dam-rock interface.

5 References

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